

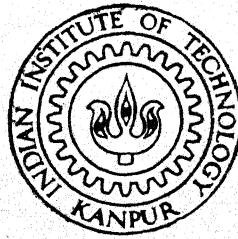
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# SUPPRESSION OF MACHINE TOOL CHATTER BY PASSIVE DAMPING

by

Lt. Sanjeev Raman

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DEPARTMENT OF MECHANICAL ENGINEERING  
**INDIAN INSTITUTE OF TECHNOLOGY KANPUR**  
MARCH, 1998

# **SUPPRESSION OF MACHINE TOOL CHATTER BY PASSIVE DAMPING**

A thesis submitted  
In partial fulfillment of the requirement  
for the degree of  
**MASTER OF TECHNOLOGY**

by

**Lt. Sanjeev Raman**

to the

**DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR**

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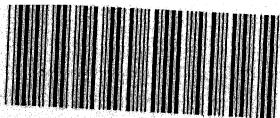
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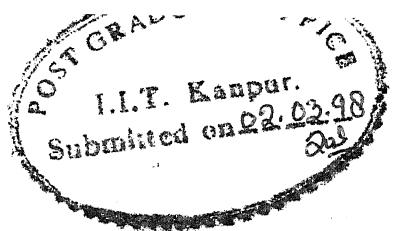
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## CERTIFICATE

It is certified that the work contained in the thesis entitled "SUPPRESSION OF MACHINE TOOL CHATTER BY PASSIVE DAMPING", by Lt. Sanjeev Raman has been carried out under my supervision and that this work has not submitted elsewhere for a degree.



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Feb. 1998

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## **ABSTRACT**

Chatter in machine tools results from a loss of stability in the cutting process due to introduction of negative damping co-efficient through an interaction of system parameters and/or directional effect of mode receptors. The need to suppress chatter arises from its detrimental effects on surface finish, productivity and machine tool life. In the present work, a simple and effective method has been suggested to improve the cutting performance by incorporating a passive type absorber in the system. The absorber consists of a spring whose stiffness can be varied to achieve maximum effectiveness depending upon the cutting conditions. Theoretical analysis, based on dynamic vibration absorber principles, shows that it improves the stability of the cutting process. Experimental investigations carried out on a conventional centre lathe, shows that the present system is capable of improving stability in certain rotational speed range. However, the absorber requires proper tuning with respect to the main system, which otherwise will result in adverse effects on the stability condition.

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# **CHAPTER 1**

## **1.1 INTRODUCTION :**

The machining of metals is often accompanied by a violent relative vibration between work and tool which is called chatter. Chatter is undesirable because of its adverse effects on surface finish, machining accuracy, and tool life. Furthermore, chatter is responsible for reducing production rate because if no remedy can be found, metal removal rates have to be lowered until the vibration free performance is obtained.

During operations, machine tools are subjected to static and/or dynamic loads, the latter having periodic or impact characteristics. These loads are associated with the cutting process and/or movements of the machine tool members. Interrelation between disturbing forces and motions and the effect on machine tool performance has been analysed by Sadowy [1] as shown in Fig.1.1. While analysing the dynamic behaviour of the machine tools, 'rigidity' and 'stability' are two important characteristics to be considered. Dynamic rigidity is a measure of the structures resistance to vibration. It refers to steady-state vibrations whereas, dynamic stability refers to transient response characteristics. The most common phenomenon of dynamic instability in machine tools is one or more forms of self excited vibrations frequently described as chatter.

The most important characteristic property of chatter vibration is that it is not induced by external forces, but rather the forces that bring it into being and maintain it, are generated by vibratory process itself as shown by Arnold [2].

Chatter is thus a self sustaining process which draws energy from an extraneous source by its own periodic motion and hence always occurs at a frequency close to the natural frequency of the undamped system.

## **1.2 REVIEW OF PREVIOUS WORK :**

Chatter in machining impedes the improvement of cutting accuracy particularly in high speed machining. Thus it becomes important to review the causes of chatter as well as the methods to suppress it. Furthermore, since chatter initiates as a result of the cutting process itself, it becomes necessary to look into the various influences of machining process upon stability conditions while trying to implement steps to avoid chatter.

### **1.2a Causes of Chatter :**

Several models have been proposed by various researchers for analysing self excited vibrations in machine tools. There are four basic concepts which by themselves alone or through mutual interaction can produce self-excited vibrations. These are schematically shown in Fig.1.2.

In the so called velocity principle, it is assumed that a component of the cutting force depends on and being in phase with the velocity of vibration, provides the sole energy for

self excited vibrations to occur. One of the earliest models based on the velocity principle is the Vander Pol's model. Here the kinetic frictional force is assumed to be a function of velocity. If the characteristic of the force of friction is negative, instability will arise and amplitude shall grow.

The theory proposed by Tobias and Fishwick [3] includes the effect of regeneration. Regeneration arises when successive cuts overlap and is caused by the variation of uncut chip thickness. Tobias model for regeneration is really an interaction between structural damping and cutting process damping terms.

Arnold [2] used the concept of velocity principle for explaining chatter in machine tools. According to Arnold, self-excited vibrations are analysed by assuming the cutting force to be a function of instantaneous cutting speed, feed, and rake angle. Experiments reveal that the cutting force, in general, decreases with speed, increases with feed and decreases with rake angle. It was also observed by Arnold [2], however, that chatter can occur even when successive cuts do not overlap each other, so that the chip-thickness variation effect is not present. Under these conditions, according to Arnold, dynamic instability is the result of the cutting force as a function of cutting speed showing a falling characteristic. But it is doubtful whether the negative damping introduced by the falling cutting force characteristics with increase in cutting speed, as suggested by Arnold [2], is capable of overcoming the positive damping of the system. Tobias [4] also showed that the negative damping introduced by a falling characteristic is not only proportional to the negative slope of the curve, but is also dependent on the dynamic behaviour of the machine

or of the tool. If it is only the tool, for example, which vibrates, this negative damping is proportional to the overhang, in other words with a large overhang even a comparatively small negative slope can have a large destabilizing effect.

Kato and Marui [5] examined the cause and mechanism of regenerative chatter vibrations due to deflection of workpiece. The regenerative chatter vibrations are induced by the phase lag of the undulations in successive cutting. This means that the small undulations initially produced on a work surface by the transient vibrations of the workpiece become larger which then extend over the whole work surface because a given amount of energy is available for exciting and maintaining the vibrations owing to the phase lag of successive undulations. In practical machining operations, it has been shown that regenerative chatter is generally more liable to occur than primary chatter (non regenerative).

The regenerative model suffers from a drawback in so far as they assumed a 'direct receptance' model of single degree of freedom system. Tlusty and Polacek [6] and even Tobias in his later publications, accepted models in which it is assumed that any mode participates not by its 'direct receptance' but by its 'cross receptance'. The directional effect is accounted for by introducing equivalent stiffness into the system. This idea has been extended to the case of forced vibration with regeneration by Tlusty and Polacek [6].

Phase lag theory was proposed by Doi and Kato [7] for explaining self excited vibrations. They showed that during cutting, the cutting force lags behind the chip thickness variation. They regarded this delay in cutting force variation as a fundamental effect and

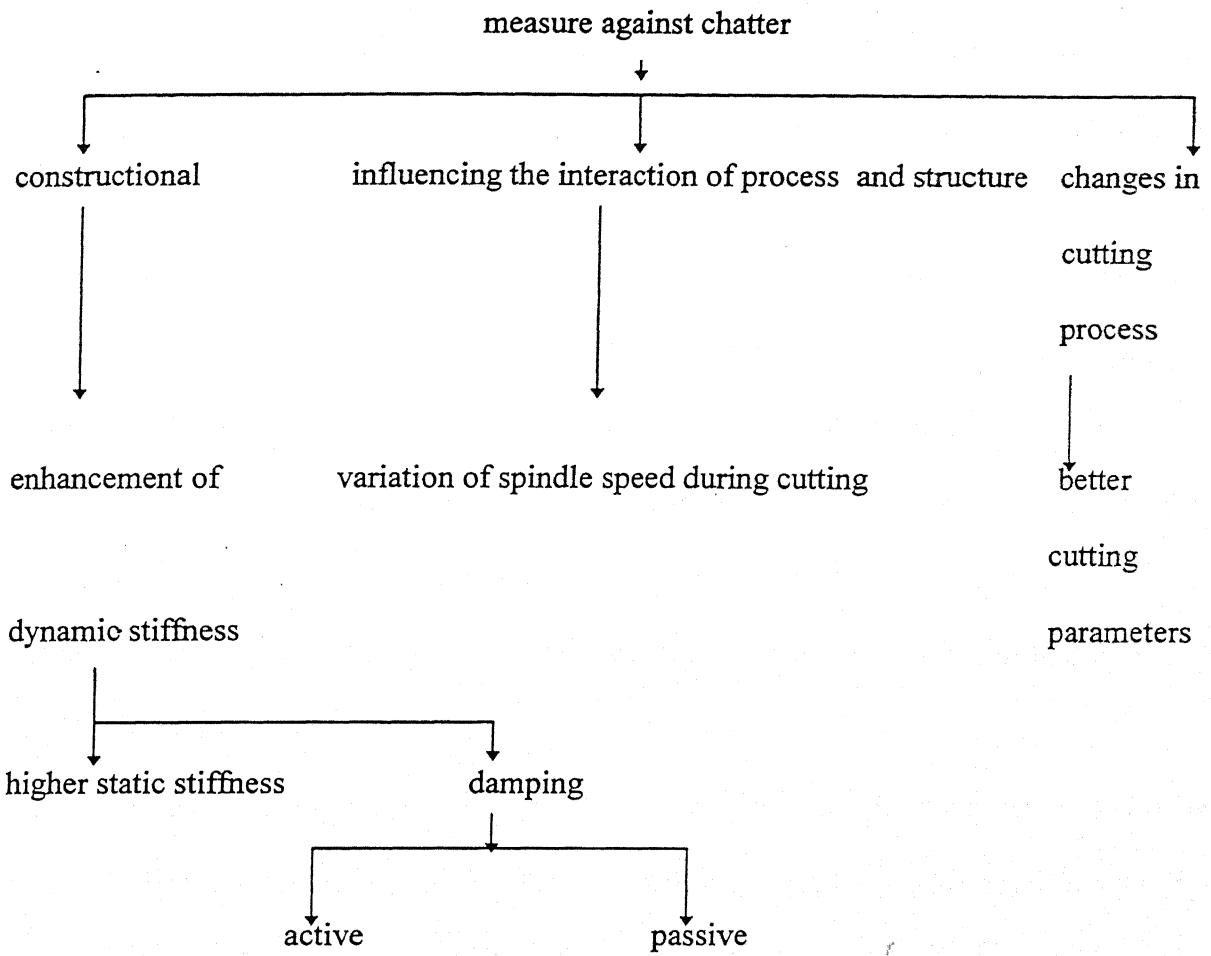
cause of chatter. Kegg [8] later found out experimentally that as clearance angle decreases, phase angle increases for both components of the cutting force.

Elyasberg [9] suggested that the frictional force along the rake face lags the tangential force. This is from the fact that after the disturbance, the tool must traverse some path relative to the workpiece material to cause a deformation which sets up stresses in the material to balance the disturbance force. Consequently, the deformation is delayed and the corresponding stresses are set up only after a certain time has passed, relative to the moment of disturbance.

Cook [10] emphasised that in the usual metal cutting operation, three processes occur simultaneously : shear on shear plane ; sliding between chip and tool; and sliding between work and tool flank. The shearing process damps vibrational motion which is parallel to the shear plane. The tool-chip interface friction force excites vibration along the tool face while the normal force may limit oscillation perpendicular to the tool face. The wear land and clearance-face friction force can excite the vibration in the vertical directions while the normal force can limit the same in the horizontal direction.

### **1.2b Elimination of Chatter :**

The measures taken to reduce chatter may be classified in the following way ::



The changes in cutting process that help in reducing chatter in machine tools are :

- (a) minimising clearance angle
- (b) negative rake angle
- (c) increasing the feed rate
- (d) choice of high or low cutting speed etc.

However these cannot be called satisfactory since the effect of these changes on the cutting process itself has to be considered as well. Minimising the clearance angle may help in reducing chatter, but increases the wear rate of the tool due to increased rubbing with the

work. Negative rake angle increases the cutting force which in turn necessitates better clamping conditions for tool and job. Moreover, increased cutting force increases the wear rate of the tool.

Continuous variation of spindle speeds is another method of suppressing chatter through the mutual interaction process. Sexton and Stone [11] suggested that under chatter conditions at constant speed, the tool oscillates at a constant frequency near a natural frequency and with a fixed critical phase, relative to the wavy surface that it is removing from the workpiece. Under these conditions, therefore, the tool is continuously excited at a frequency near the natural frequency of the system. A stabilising effect from variable speed might arise in two ways :

- (a) The phase angle between the vibration of the tool and the wave on the workpiece surface will continuously be changing and the "critical" phase angle that leads to instability will rarely be achieved.
- (b) The waves on the workpiece surface will be removed at a speed different from that at which they were left by the tool, one revolution previously, and thus the tool will not be excited at a constant frequency close to the natural frequency but at a continuously varying frequency.

However, the experimental results [11] have shown the presence, even under stable conditions, of large transient vibrations which, in practice, reduce significantly the effectiveness of varying spindle speed. Therefore, the improvements in stability resulting from the use of a continuously varying spindle speed are only modest.

Of all the methods available, it is the enhancement of dynamic stiffness through constructional efforts which is most effective [12]. Static stiffness cannot be increased without modifying the structure. Therefore introducing damping either through active or passive means helps to improve stability greatly. The main source of damping in machine tools are the structural damping because of molecular structure of the machine tool and frictional damping in which energy is dissipated due to the rubbing of the surfaces of two elements at the interface. Frictional damping can be increased by using welded joints. However, a large amount of damping is desirable not only from the chatter viewpoint, but also for the purpose of absorbing free or forced vibrations.

The dynamic absorbers improve the resonance of the primary system by adding an auxiliary mass to the main system. This shifts the resonant frequency of the mechanical system away from the operating frequency of the vibratory force. Whereas the basic principle of passive vibration absorber depends on the principle of transferring vibrational energy to an auxiliary damper system. Tuning of this secondary mass with the primary mass is very important since mismatching can produce adverse results. Active dampers are effective over a wide range of varying conditions. The fundamental requirement here is to apply a force to the main system to resist its motion. At resonance, such a force should be in antiphase with the velocity of the point of application which is achieved with the use of a control system.

Cowley and Boyle [13] presented an analysis of active dampers based on electrodynamic arrangement. The electrodynamic generator produces a force proportional to the instantaneous signal produced from the transducer mounted on the structure. They

showed that the electrodynamic vibrator controlled by velocity feedback is capable of providing additional damping proportional to the feedback gain.

Seto [14] proposed a new design method for a variable stiffness type dynamic absorber with viscous damping devised to increase the cutting performance of machine tools with long overhung rams as in vertical lathes and boring machines. The absorber has a frequency tuner which is tuned to maximum efficiency over the interested frequency range.

Kim and Ha [15] designed a viscoelastic damper which is attached to the tool post of the lathe to suppress chatter. Because one of the resonance frequencies responsible for the chatter varies depending upon the location of the carriage on the sliding surface, the prestrain of the viscoelastic element, which is initially optimum tuned and damped at a location of the carriage, is readjusted for optimum tuning at other locations.

Rivin and Kang [16] proposed that by adding a damping element in series with the cutting tool and by judicious choice of the stiffness of this element, the stability of the system can be increased even though the actual stiffness of the cutting tool is reduced.

### **1.3 OBJECTIVES OF THE PRESENT WORK :**

In steady state machining the cutting force depends on the nature of the tool work interface, the feed rate, cutting speed and depth of cut. Under experimental conditions, the

parameters stated above do not remain constant but vary about a mean value in a random manner. Due to this vibratory nature of the cutting forces on the tool, the resulting compliance also varies depending upon the frequencies of vibrations. Dimensional inaccuracies are direct consequence of this variation of resulting compliance, and therefore, it is necessary to reduce them through simple and effective means.

In the present work, a passive-type damper, whose stiffness can be varied, is designed and attached to a conventional lathe. Higher cutting stiffness and hence higher stability can be obtained by incorporating such absorbers to the tool post. The addition of the absorber not only reduces the compliance of the main system, but also through a proper choice of parameters it is possible to increase the effective damping in the main system leading to improved machine tool stability.

Thus the objectives of the present work can be stated briefly as :

- (a) To design a passive-type vibration absorber for the lathe tool to reduce chatter.
- (b) To carry out theoretical analysis based on established chatter theory for proper design of the absorber.
- (c) Experimentally investigate the proposed system.

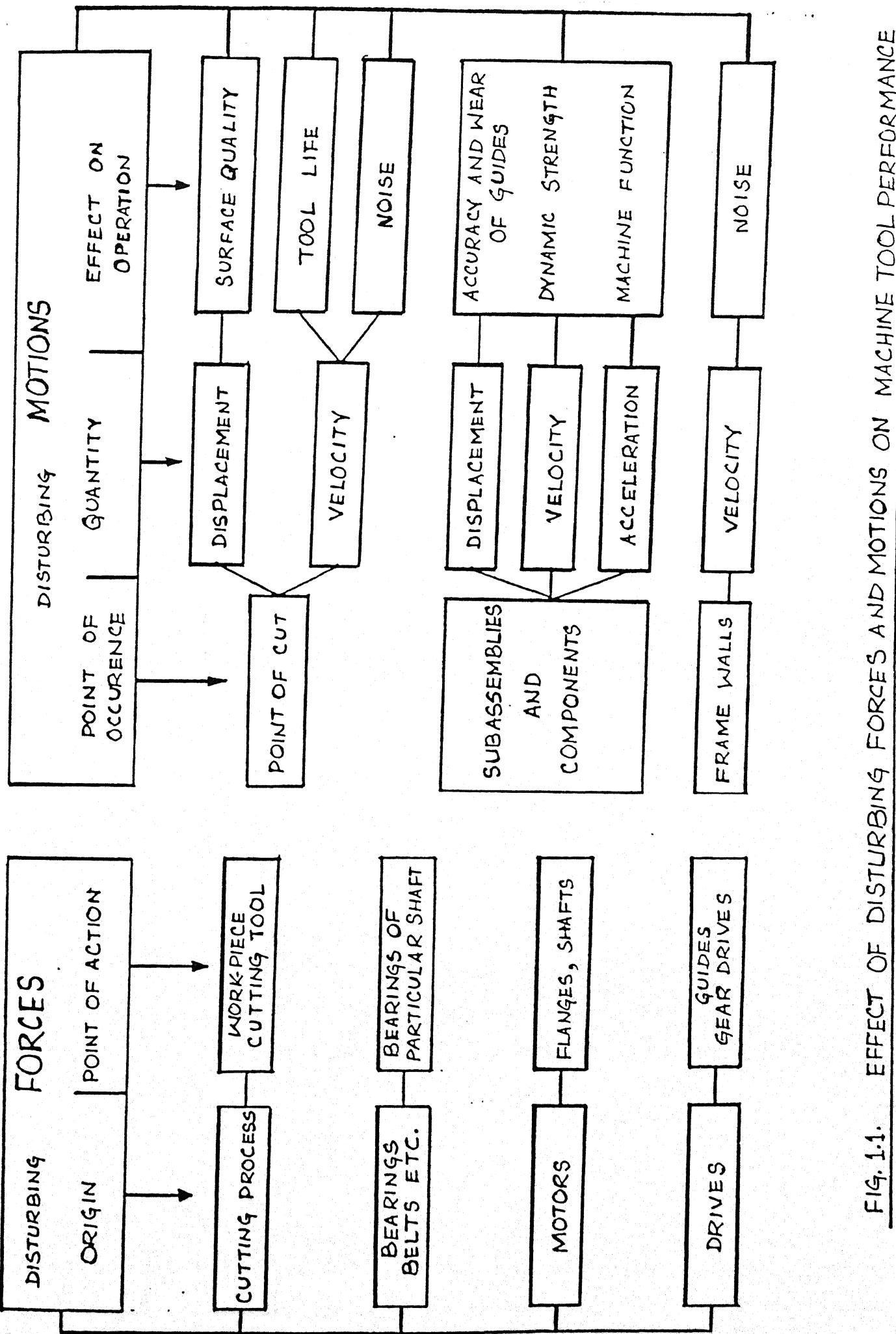


FIG. 1.1. EFFECT OF DISTURBING FORCES AND MOTIONS ON MACHINE TOOL PERFORMANCE

## THEORY OF CHATTER

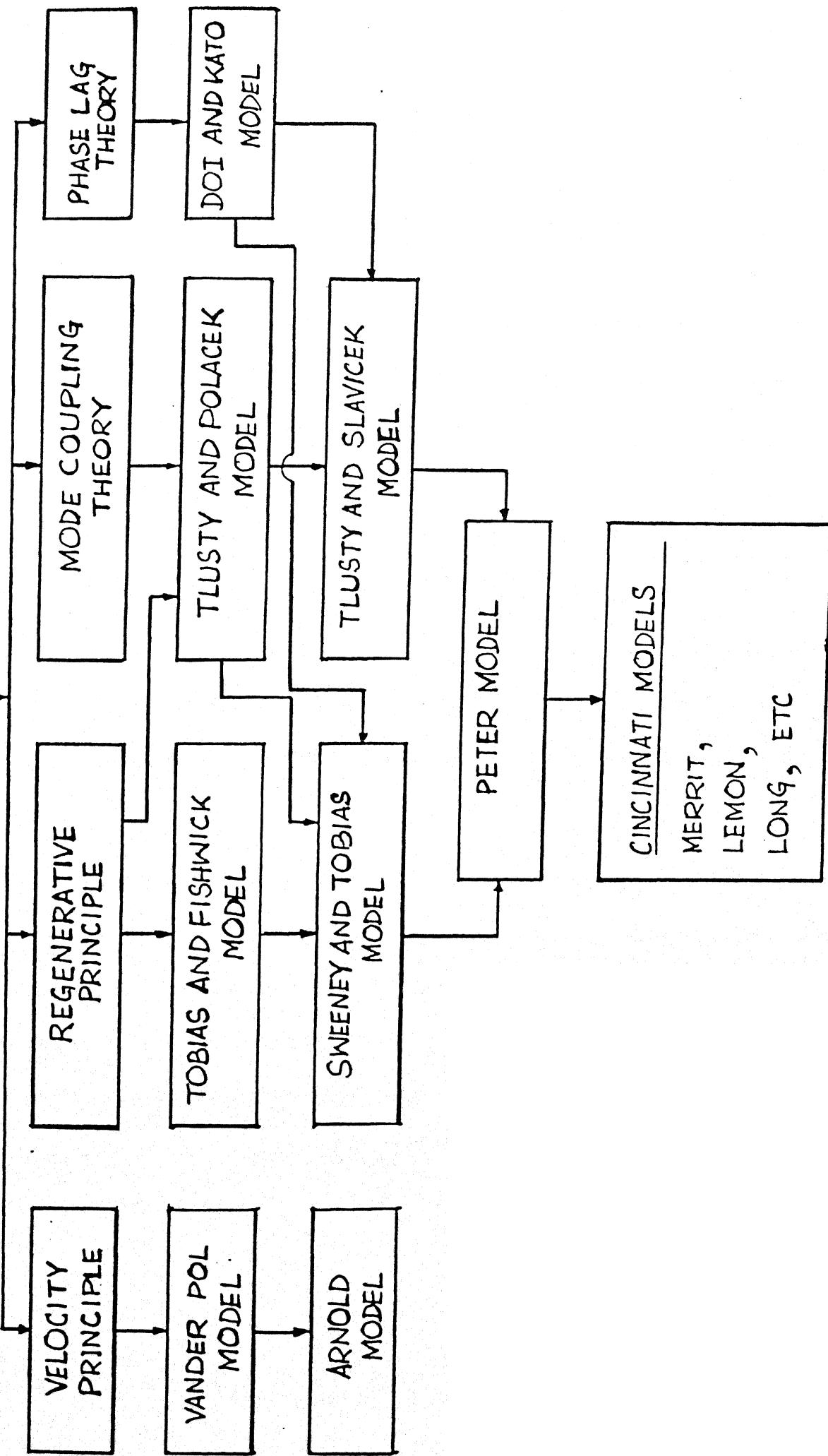


FIG. 1.2: INTERRELATION BETWEEN DIFFERENT THEORIES AND MODELS

## CHAPTER II

### 2.1 STABILITY OF TWO DEGREE OF FREEDOM SYSTEM :

The analysis has been carried out under the following assumptions :

- (a) The cutting is orthogonal, that is, the cutting force, the job motion and the tool motion can be represented in one plane (xoy).
- (b) The tool is sharp.
- (c) There is no rubbing along the tool clearance face.
- (d) The tool is infinitely stiff in y direction (Refer Fig. 2.1).
- (e) The chatter is of type A, that is, the relative chatter amplitudes of the cutting edge are in line with the tool shank (Refer Fig. 2.1).

#### 2.1a The Dynamic Cutting Process : [4]

The conventional theory of metal cutting deals entirely with steady-state cutting process in which the cutting takes place under vibration-free cutting conditions. The chatter theory deals with metal cutting under non-steady state conditions, that is, in the presence of vibrations. The difference between static and dynamic cutting processes is easily recognized as shown in Fig.2.2.

Under steady state conditions :

- (a) Chip thickness 'S' is equal to feed  $S_o$ [mm/rev]

(b) Cutting speed  $V_o$ [mm/s] is constant and  $V_o=R\Omega=2\pi RN$  where  $R$  is the radius of the work and  $N=\Omega/2\pi$  is the rotational speed in revolutions per second. ( $\Omega=2\pi/T$  is the angular velocity of the tool.  $T$  is the time for one revolution,  $N=1/T$ ).

(c) The feed rate  $r_o=S_o\Omega/2\pi=S_oN$  [mm/s] and hence it is sufficient to give  $S_o$  and  $V_o$  (that is  $\Omega$ ) because these decide feed rate,  $r_o$ . The only independent variables are therefore  $S_o$  and  $\Omega$ . For these reasons the cutting force  $P_o$ , is regarded for the purpose of static investigations as a function of  $S_o$  and  $V_o$ , that is  $P_o=f(S_o, V_o)$ .

The dynamic cutting conditions : let us now assume that the tool strikes a hard grain 'A' in the material as a result of which the cutting force  $P_o$  is suddenly increased by  $dP$ . Force  $P_o$ , however has already been absorbed by the static deformation of the machine frame. The increase in the cutting force gives rise to further deformation.

If the hard grain now breaks out at time  $t=0$ , a sudden drop in cutting force and torque will occur, and consequently the potential energy stored in the machine frame and drive will be released to throw the system into vibration. Consequently, the hard grain encountered changes the cutting force conditions. Let us first examine the dynamic cutting process encountered at time  $t=0$  and continuing till the vibration has completely decayed. At time  $t=0$ , the cutting edge of the tool starts to move along a damped vibration curve superimposed on the steady state cutting path as indicated in Fig. 2.2(b) by curve 1. It is obvious that the instantaneous chip thickness 'S' is no longer the same as the nominal field ' $S_o$ '. The instantaneous chip thickness is therefore  $S = S_o + dS$ , where  $dS$  and  $S$  are time dependent factors. The quantity  $dS$  is the **Chip Thickness Variation**.

Similarly, the feed rate also becomes time dependent so that  $r = r_o + dr = S_o N + dr$ . The quantity  $dr$  is the feed rate variation.

If the work undergoes torsional vibration also, the cutting speed likewise will vary to give

$$V = V_o + dV = V_o + 2\pi RN.$$

It is clear that under dynamic cutting conditions  $dS$ ,  $dr$  and  $dV$  and hence  $S$ ,  $r$  and  $V$  are independent of each other, so that the operation for the cutting force contains three independent parameters, that is,  $P = f(S, r, \Omega)$ .

For chatter theory, it is necessary to examine how the cutting forces change when small variation in these factors take place. Under static conditions, the cutting force variation is defined by

$$dP_o = K_s dS_o + K_v dV_o = K_s dS_o + K_\Omega d\Omega \quad (2.1)$$

The two parameters  $K_s$  and  $K_\Omega = RK_v$  are easily found for any given cutting process.

Mathematically

$$K_s = (\partial P_o / \partial S_o)_{dv=0} \quad (2.2)$$

$$K_\Omega = (\partial P_o / \partial \Omega_o)_{ds=0} \quad (2.3)$$

To find  $K_s$ , it is therefore necessary to find cutting force as a function of chip thickness for constant speed. Similarly to find  $K_\Omega$ , it is necessary to find the characteristic of cutting force as a function of cutting speed for constant feed.

Under dynamic cutting conditions, the cutting force variation for a small change in these factors is given by

$$dP = K_1 dS + K_2 dr + K_3 d\Omega \quad (2.4)$$

Again, it will be seen that mathematically

$$K_1 = (\partial P / \partial S) \quad dr = d\Omega = 0 \quad (2.5)$$

$$K_2 = (\partial P / \partial r) \quad dS = d\Omega = 0 \quad (2.6)$$

$$K_3 = (\partial P / \partial \Omega) \quad dS = dr = 0 \quad (2.7)$$

$K_1$ ,  $K_2$  and  $K_3$  are dynamic coefficients which are to be determined by experiments. For constant speed case :  $d\Omega = 0$

Therefore  $dP = K_s dS = K_1 dS + K_2 N dS$

$$\text{or} \quad K_2 = (K_s - K_1)/N \quad (2.8)$$

Similarly for  $dS=0$  and  $d\Omega \neq 0$

$$dP = K_1 dS + K_2 dr + K_3 d\Omega = K_\Omega d\Omega$$

$$\text{or} \quad K_3 = (K_\Omega - K_2 S) = K_\Omega - (K_s - K_1) S_0 / \Omega \quad (2.9)$$

Equations (2.4) can therefore be expressed in terms of the coefficients of static equation as

$$dP = K_1 dS + (K_s - K_1)/N dr + \left[ K\Omega - (K_s - K_1) \frac{S_o}{\Omega} \right] d\Omega \quad (2.10)$$

The equation (2.10) therefore has only one dynamic coefficient  $K_1$  which is **chip thickness coefficient**. The quantity  $(K_s - K_1) = K_R$  is designated as the **penetration coefficient**, whilst  $[K\Omega - (K_s - K_1) S_o/\Omega]$  is termed as the **cutting speed coefficient**. Therefore under dynamic conditions, there are three physical parameter which may vary independently of each other and which alongwith their respective coefficients determine the dynamic cutting force variation  $dP$  acting on the tool. It is seen therefore that the physical cause of chatter may be divided into three groups depending upon whether they lead to dynamic instability through influencing

- (a) Chip thickness variation,  $S$
- (b) The rate of penetration,  $r$
- (c) The rotational speed,  $\Omega$

### 2.1b The Physical Cause of Chatter :

When a steady-state cutting operation is disturbed, the cutting force element 'dP' is generated. It may so happen that  $dP$  is of such a form that it increases the original disturbance so that a still larger  $dP$  is set up, and so on. The system under these conditions begins to vibrate and becomes unstable. On the other hand,  $dP$  also may act

counter to the disturbance so that the original disturbance vanishes and steady-state cutting is resumed. Under these conditions, the system is stable.

The disturbance affecting the cutting process is time dependent, and therefore the cutting force element  $dP$  is also a function of time. If chatter occurs, however, the time dependence of  $dP$  will not alone give rise to instability. The important property of  $dP$  is that it depends not only on the displacement brought about by the disturbance but also on its velocity. Forces which are velocity dependent can be regarded as damping forces, and consequently they may either add to or subtract from the damping forces contained in the system. If the damping introduced by  $dP$  is positive, it will increase the ordinary damping present with the result that the prevailing disturbance will decay fast. On the other hand, if the damping force brought about by the disturbance is negative, it will reduce the damping of the system. Positive damping is energy absorbing whereas negative damping introduces energy into the system which is used to build up vibration and maintain it.

### 2.1c Chatter in Lathes : [12]

In lathe machines the equivalent stiffness of the machine body is much more than the equivalent stiffness of the tool and job support together. Therefore, the chatter mainly remains confined in the cutting tool support and job support and the characteristic of the structure of lathe body as a whole has been excluded from being considered.

The single degree of freedom chatter theory will be considered for only those cases where rigidity of the tool and support is relatively small in one direction, so as to allow the tool to vibrate in one direction only. Otherwise the tool motion will not be straight and two degree of freedom theory will have to be used for analysing the problem. For the purpose of studying dynamic behaviour, the machine body frame is replaced by an elementary vibratory system.

Consider a workpiece-tool system under the action of  $dPx$  as shown in Fig. 2.3. It is assumed that the steady-state cutting process has been disturbed by a relative vibration,  $x$ , occurring between work and tool and having the direction of a principal vibration of type A.

The differential equation of motion for direct receptance is

$$mD^2x + cDx + Kx = -dPx \quad (2.11)$$

Where,  $m$  = mass of the tool system

$c$  = equivalent damping of the tool system

$K$  = equivalent stiffness of the tool system

$x$  = vibratory displacement of the tool.

$dPx$  = cutting force element in the  $x$  direction

It is assumed that the dominant mode of vibration is in  $x-x$  direction, and there is no incremental change in rotational speed, that is  $d\Omega=0$ .

Hence

$$mD^2x + cDx + Kx = -[K_1dS + K_r / N \ dr] \quad (2.12)$$

Now  $dS = x(t) - \mu x(t-T)$  (2.13)

Where

$T$  is the time required by the job to complete one revolution and is equal to  $1/N$ .

$x(t)$  denotes the vibratory displacement of the tool at any instant from its mean position.

$\mu$  is the over lapping factor

Further  $dr = Dx$  (2.14)

Assuming the solution to be,

$$x = A e^{\alpha t} \cos \omega t$$

Where

$A$  = amplitude

$\alpha$  = phase angle decay const.

$\omega$  = chatter frequency

We have,

$$\begin{aligned} dS &= A e^{\alpha t} \cos \omega t - \mu A e^{\alpha(t-T)} \cos \omega(t-T) \\ &= F_1 x + F_2 Dx \end{aligned} \quad (2.15)$$

Where

$$F_1 = 1 - \mu e^{-\alpha T} [\cos \omega T + (\alpha/\omega) \sin \omega T] \quad (2.16)$$

$$F_2 = (\mu/\omega) e^{-\alpha T} \sin \omega T \quad (2.17)$$

Substituting equation (2.14) and (2.15) in equation (2.12), we have

$$mD^2x + cDx + Kx = -K_1 [F_1x + F_2Dx] - (K_R/N) Dx$$

$$\text{Or } mD^2x + [c + K_1 F_2 + K_R/N]Dx + (K + K_1 F_1)x = 0 \quad (2.18)$$

We know that

$$\begin{aligned} \text{Natural frequency } \omega_0^2 &= K/m ; \text{ effective dynamic amplification } Q_e, \text{ at resonance,} \\ &= 1/2D = \sqrt{Km}/c \end{aligned}$$

$$\text{and } c/k = 1/Q_e \omega_0$$

Substituting the aforesaid terms in above equation, we get

$$\left[ \frac{1}{\omega_0^2} \right] D^2x + \left[ \frac{1}{Q_e \omega_0} + \frac{K_1 F_2}{K} + \frac{K_r}{KN} \right] Dx + \left[ 1 + \frac{K_1 F_1}{K} \right] x = 0 \quad (2.19)$$

Since  $x = A e^{\alpha t} \cos \omega t$ , obtaining corresponding  $Dx$  and  $D^2x$  terms and grouping the trigonometric terms, the following equations are obtained

$$\left[ \frac{\alpha^2}{\omega_0^2} \right] - \frac{\omega^2}{\omega_0^2} + \alpha \left[ \frac{1}{Q_e \omega_0} + \frac{K_1 F_2}{K} + \frac{K_r}{KN} \right] + \left[ 1 + \frac{K_1 F_1}{K} \right] = 0 \quad (2.20)$$

$$-\frac{2\alpha^2}{\omega_0^2} - \left[ \frac{1}{Q_e \omega_0} + \frac{K_1 F_2}{K} + \frac{K_r}{KN} \right] = 0 \quad (2.21)$$

For threshold stability  $\alpha=0$ , and substituting the values of the  $F_1$  and  $F_2$  from eq. (2.16) and (2.17), equations (2.20) and (2.21) reduce to

$$(\omega/\omega_0) = 1 + \frac{K_1}{K} [1 - \mu \cos(\omega/N)] \quad (2.22)$$

$$\text{and } \frac{1}{Q_e \omega_0} + \left( \mu \frac{K_1}{\omega K} \right) \sin(\omega / N) + \frac{K}{KN} = 0 \quad (2.23)$$

Equation (2.22) determines the chatter frequency at the stability threshold whilst Eq.(2.23) express the requirements for zero overall damping. If in Eq. (2.23) effective dynamic amplification factor  $Q_e$  is replaced by effective damping  $D_e$  such that  $D_e = \frac{1}{2} Q_e$ , then equation (2.23) can be expressed as a sum of three factors :

$$(a) \text{ Effective damping ( frame damping + chip friction) } D_e \quad (2.24)$$

$$(b) \text{ Damping due to variation in chip thickness } D_s$$

$$D_s = \frac{\mu K_1}{2 \omega K} \sin(\omega / N), \text{ and} \quad (2.25)$$

$$(c) \text{ Damping due to variation in rate of penetration } D_r$$

$$D_r = \frac{K_r}{2KN} \quad (2.26)$$

If  $d\Omega$  is not taken to be zero, then damping due to variation in cutting speed can also be included which is  $D_v$

$$(d) \text{ and } D_v = \frac{K_\Omega}{2KR} = \frac{1}{2} \frac{K_v}{K} \quad (2.27)$$

The effective damping is always positive (energy-absorbing). The other damping effect, however, may be positive or negative or nil and if the total damping, that is  $D_e + D_s + D_r$  is negative, the cutting process will be unstable.

$$\text{Combined Damping } D_c = D_e + D_s + D_r + D_v \quad (2.28)$$

$$\text{Let } B = \frac{1}{Q_e} + \frac{K_r}{KN} \omega_0 \quad (2.29)$$

then equation (2.23) can be rewritten as

$$\frac{\omega}{\omega_0} B = -\mu \frac{K_1}{K} \sin(\omega / N) \quad (2.30)$$

squaring equation (2.30) and dividing by eq. (2.22) gives

$$B^2 = \frac{\mu^2 (K_1 / K)^2 \sin^2(\omega / N)}{1 + (K_1 / K)[1 - \mu \cos(\omega / N)]} \quad (2.31)$$

cross multiplying and rearranging, we get

$$\mu^2 \frac{K_1^2}{K^2} \cos^2 \omega / N - \mu B^2 \frac{K_1}{K} \cos \omega / N + \left( B^2 + B^2 \frac{K_1}{K} - \mu^2 \frac{K_1^2}{K^2} \right) = 0 \quad (2.32)$$

The solution of above is

$$\cos \omega / N = \frac{1}{2} \left[ \frac{B^2 K}{K_1} \pm \sqrt{\left( \frac{B^2 K}{K_1} \right)^2 - \frac{4 B^2 K^2}{K_1^2} \left( 1 + \frac{K_1}{K} \right) + 4} \right] \quad (2.33)$$

The solution of  $\cos \omega / N$  must satisfy two conditions

(a) it must be a real number

(b) it is necessary that  $|\cos(\omega / N)| \leq 1$

If  $B$  is regarded as function of  $Q_e$ , there will be a range of  $Q_e$  values for which equation (2.32) will have two real roots. It is important, however, not to go below a minimum value of  $Q_e = Q_m$ , otherwise these roots will be complex numbers. Corresponding to this

value of  $Q_m$  is  $B=B_m$ , and  $B_m$  is obtained from the condition that the expression under the root sign in equation (2.33) is equal to zero.

That is,

$$\frac{B_m^4 K^2}{K_1^2} - \frac{4B_m^4 K^2}{K_1^2} \left(1 + \frac{K_1}{K}\right) + 4 = 0 \quad (2.34)$$

Solving yields,

$$B_m^2 = 2 \left[ 1 + \frac{K_1}{K} \pm \sqrt{1 + 2 \frac{K_1}{K}} \right] \quad (2.35)$$

The question whether a positive or negative sign is permissible for the root is decided by the second condition which must be satisfied by the solutions of equations (2.32), namely that  $|\cos(\omega/N)| < 1$ ; for, if  $Q_e = Q_m$  and hence  $B=B_m$ , equation (2.33) becomes

$$\cos(\omega/N) = \frac{1}{2} B_m^2 \frac{K}{K_1} = \frac{K}{K_1} \left[ 1 + \frac{K_1}{K} \pm \sqrt{1 + 2 \frac{K_1}{K}} \right] \quad (2.36)$$

From above, it is clear that only if the sign is negative can the condition  $|\cos(\omega/N)| < 1$  be satisfied. On substituting the value of  $B$  from Eq. (2.29) in Eq. (2.36), the minimum value of  $Q_e$ , that is,  $Q_m$ , for which oscillation is possible is given as

$$Q_m = \frac{1}{\sqrt{\left\{ 2 \left[ 1 + \frac{K_1}{K} - \sqrt{1 + 2 \frac{K_1}{K}} \right] \right\} - \frac{K_r \omega_o}{KN}}} \quad (2.37)$$

Thus for every rotational speed,  $N$ , there is a minimum  $Q_m$ .

## 2.1d Equations of motion with absorber : [17]

In Fig. 2.4,  $m, c$ , and  $K_a$  are parameter of machine tool and are therefore fixed values; whereas  $m, c$ , and  $K_b$  are parameter of the vibration absorber and are adjustable subject to certain practical considerations. Let us define  $m_2 = \lambda m_1$  where  $\lambda$  is the order of 0.1.

The equation of motion of the main mass is

$$m_1 D^2 x + c_1 D x + K_a x = c_2 D(y-x) + K_b(y-x) - dPx$$

$$\text{or } [m_1 D^2 + (c_1 + c_2) D + (K_a + K_b)]x = [c_2 D + K_b]y - dPx \quad (2.38)$$

The equation of motion of the absorber is

$$m_2 D^2 y + c_2 D(y-x) + K_b(y-x) = 0$$

$$\text{or } [m_2 D^2 + c_2 D + K_b]y = [c_2 D + K_b]x \quad (2.39)$$

Eliminating  $y$  from Eq. (2.38) and (2.39) gives

$$[m_1 D^2 + (c_1 + c_2) D + (K_a + K_b)]x - \frac{[c_2 D + K_b][c_2 D + K_b]x}{[m_2 D^2 + c_2 D + K_b]} = -dPx$$

Cross multiplying we get,

$$\begin{aligned} \text{or } & [m_1 m_2 D^4 + \{m_2 c_1 + (m_1 + m_2) c_2\} D^3 + \{c_1 c_2 + (m_1 + m_2) K_b + m_2 K_a\} D^2 \\ & + (c_2 K_a + c_1 K_b) D + K_a K_b]x = -[m_2 D^2 + c_2 D + K_b] dPx \end{aligned} \quad (2.40)$$

Writing  $\omega_1^2 = K_a / m_1$        $c_1 / K_a = 1 / (Q_1 \omega_1)$

$$\omega_2^2 = K_b / m_2 \quad c_2 / K_b = 1 / (Q_2 \omega_2)$$

where

$\omega_1$  = natural frequency of machine tool.

$\omega_2$  = natural frequency of the absorber.

or Dividing throughout by  $K_a K_b$ , we have

$$\left[ \frac{D^4}{\omega_1^2 \omega_2^2} + \left\{ \frac{1}{\omega_2^2 \omega_1 Q_1} + \frac{(1+\lambda)}{\omega_1^2 Q_2 \omega_2} \right\} D^3 + \left\{ \frac{1}{\omega_1 \omega_2 Q_1 Q_2} + \frac{(1+\lambda)}{\omega_1^2} + \frac{1}{\omega_2^2} \right\} D^2 \right. \\ \left. + \left\{ \frac{1}{\omega_2 Q_2} + \frac{1}{\omega_1 Q_1} \right\} D + 1 \right] x = \\ - \left[ \frac{D^2}{\omega_2^2} + \frac{D}{\omega_2 Q_2} + 1 \right] \left[ \frac{K_1 F_1}{K_a} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) D \right] x$$

or LHS =

$$- \left[ \frac{K_1 F_1}{K_a \omega_2^2} D^2 + \frac{K_1 F_1}{K_a \omega_2 Q_2} D + \frac{K_1 F_1}{K_a} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{1}{\omega_2^2} D^3 + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{D^2}{\omega_2 Q_2} + \right. \\ \left. \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) D \right] x$$

$$\begin{aligned}
 & \left[ \frac{D^4}{\omega_1^2 \omega_2^2} + \left\{ \frac{1}{\omega_2^2 \omega_1 Q_1} + \frac{(1+\lambda)}{\omega_1^2 Q_2 \omega_2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{1}{\omega_2^2} \right\} D^3 \right. \\
 \text{or } & \left. + \left\{ \frac{1}{\omega_1 \omega_2 Q_1 Q_2} + \frac{(1+\lambda)}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{K_1 F_1}{K_a \omega_2^2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{1}{\omega_2 Q_2} \right\} D^2 \right. \\
 & \left. + \left\{ \frac{1}{\omega_2 Q_2} + \frac{1}{\omega_1 Q_1} + \frac{K_1 F_1}{K_a \omega_2 Q_2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \right\} D + \left\{ 1 + \frac{K_1 F_1}{K_a} \right\} \right] x = 0
 \end{aligned}$$

Assuming the same solution for  $x$  as in earlier case,

we have

$$x = A e^{\alpha t} \cos \omega t$$

$$Dx = A \alpha e^{\alpha t} \cos \omega t - A \omega e^{\alpha t} \sin \omega t$$

$$D^2x = A \alpha^2 e^{\alpha t} \cos \omega t - A \omega^2 e^{\alpha t} \cos \omega t - 2A \omega \alpha e^{\alpha t} \sin \omega t$$

$$D^3x = A \alpha^3 e^{\alpha t} \cos \omega t - A \alpha^2 \omega e^{\alpha t} \sin \omega t$$

$$D^4x = A \alpha^4 e^{\alpha t} \cos \omega t - A \alpha^2 \omega^2 e^{\alpha t} \cos \omega t - 2A \omega \alpha^3 e^{\alpha t} \sin \omega t$$

Substituting the above value of  $D^4x$ ,  $D^3x$ ,  $D^2x$ ,  $Dx$  and  $x$  and with necessary grouping,

we have,

$$e^{\alpha t} \cos \omega t \left[ \left( \frac{\alpha^4}{\omega_1^2 \omega_2^2} - \frac{\alpha^2 \omega^2}{\omega_1^2 \omega_2^2} \right) + \left\{ \frac{1}{\omega_2^2 \omega_1 Q_1} + \frac{(1+\lambda)}{\omega_1^2 Q_2 \omega_2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{1}{\omega_2^2} \right\} \alpha^3 \right. \\ \left. + \left\{ \frac{1}{\omega_1 \omega_2 Q_1 Q_2} + \frac{(1+\lambda)}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{K_1 F_1}{K_a \omega_2^2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{1}{\omega_2 Q_2} \right\} \alpha^2 \right. \\ \left. + \left\{ \frac{1}{\omega_2 Q_2} + \frac{1}{\omega_1 Q_1} + \frac{K_1 F_1}{K_a \omega_2 Q_2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \right\} \alpha + \left\{ 1 + \frac{K_1 F_1}{K_a} \right\} \right. \\ \left. - \omega^2 \left\{ \frac{1}{\omega_2 Q_2} \frac{1}{\omega_1 Q_1} + \frac{1+\lambda}{\omega_1^2} + \frac{K_1 F_1}{K_a \omega_2^2} + \left( \frac{K_1 F_2}{K_a \omega_2 Q_2} + \frac{K_r}{K_a N Q_2 \omega_2} \right) \right\} \right] = 0$$

(2.41)

and

$$e^{\alpha t} \sin \omega t \left[ \left( -\frac{2\alpha^3 \omega}{\omega_1^2 \omega_2^2} \right) - \left\{ \frac{1}{\omega_2^2 \omega_1 Q_1} + \frac{(1+\lambda)}{\omega_1^2 Q_2 \omega_2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{1}{\omega_2^2} \right\} \omega \alpha^2 \right. \\ \left. - \left\{ \frac{1}{\omega_1 \omega_2 Q_1 Q_2} + \frac{(1+\lambda)}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{K_1 F_1}{K_a \omega_2^2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \frac{1}{\omega_2 Q_2} \right\} 2\omega \alpha \right. \\ \left. - \left\{ \frac{1}{\omega_2 Q_2} + \frac{1}{\omega_1 Q_1} + \frac{K_1 F_1}{K_a \omega_2 Q_2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \right\} \omega \right] = 0$$

(2.42)

At stability threshold  $\alpha = 0$ , and substituting the values of  $F_1$  and  $F_2$  from Eq. (2.16) and (2.17), Eq. (2.41) and (2.42) reduces to

$$\left\{ \frac{1}{\omega_2 Q_2} + \frac{1}{\omega_1 Q_1} + \frac{K_1 F_1}{K_a \omega_2 Q_2} + \left( \frac{K_1 F_2}{K_a} + \frac{K_r}{K_a N} \right) \right\} = 0$$

or  $\frac{1}{\omega_1 Q_1} + \frac{1}{\omega_2 Q_2} + \frac{K_1(1 - \mu \cos \omega / N)}{K_a \omega_2 Q_2} + \frac{K_1 \mu \sin \omega / N}{K_a \omega} + \frac{K_r}{K_a N} = 0 \quad (2.43)$

and

$$\begin{aligned} & \left[ \omega^2 \left\{ \frac{1}{\omega_2 Q_2} \frac{1}{\omega_1 Q_1} + \frac{1 + \lambda}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{K_1(1 - \mu \cos \omega / N)}{K_a \omega_2^2} + \left( \frac{K_1 \mu \sin \omega / N}{\omega K_a \omega_2 Q_2} + \frac{K_r}{K_a N Q_2 \omega_2} \right) \right\} \right] \\ &= \left[ \frac{1 + K_1(1 - \mu \cos \omega / N)}{K_a} \right] \end{aligned} \quad (2.44)$$

Now let us assume that the viscous damping present in the absorber is very small. For above consideration, let us examine Eq. (2.43) which gives the requirement for zero overall damping.

As  $c_2 \rightarrow 0 ; 1/Q_2 \rightarrow 0$

Eq. (2.43) then becomes

$$\frac{1}{Q_1} + \frac{K_1 \omega_1 \mu \sin \omega / N}{K_a \omega} + \frac{K_r \omega_1}{K_a N} = 0$$

Equation (2.45) is comparable to Eq. (2.23) obtained earlier without the absorber.

Let us find the effective amplification factor of the main mass, which is  $Q_1$  in this case, to find its implication on damping of the main mass.

$$\text{Let } B = \frac{1}{Q_1} + \frac{K_r \omega_1}{K_a N} \quad (2.46)$$

Then Eq. (2.45) becomes

$$\omega B / \omega_1 = -\frac{K_1 \mu \sin \omega / N}{K_a} \quad (2.47)$$

Using the above condition, Eq. (2.44) reduces to

$$\left[ \omega^2 \left\{ \frac{1+\lambda}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{K_1(1-\mu \cos \omega / N)}{K_a \omega_2^2} + \right\} \right] = \left[ \frac{1+K_1(1-\mu \cos \omega / N)}{K_a} \right] \quad (2.48)$$

Squaring Eq. (2.47) and dividing by Eq. (2.48) we get,

$$\frac{B^2}{\left[ \omega^2 \left\{ \frac{1+\lambda}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{K_1(1-\mu \cos \omega / N)}{K_a \omega_2^2} + \right\} \right]} = \frac{\mu^2 \left( \frac{K_1}{K_a} \right)^2 \sin^2(\omega / N)}{1 + \frac{K_1}{K_a}(1 - \mu \cos \omega / N)}$$

$$B^2 = \frac{\mu^2 \left( \frac{K_1}{K_a} \right)^2 \sin^2(\omega / N)}{1 + \frac{K_1}{K_a}(1 - \mu \cos \omega / N)} \left[ \omega^2 \left\{ \frac{1+\lambda}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{K_1(1-\mu \cos \omega / N)}{K_a \omega_2^2} + \right\} \right] \quad (2.49)$$

From Fig. 2.5, equating opposite vertical sides of the two vector diagram, we have

$$\frac{c_2 K_a + c_1 K_b}{m_2 c_1 + (m_1 + m_2) c_2} = \omega^2 = \frac{K_b}{m_2}$$

Cross multiplying and simplifying yields

$$\frac{m_1 + m_2}{K_a} = \frac{m_2}{K_b} \quad \text{which is independent of both } c_1 \text{ and } c_2$$

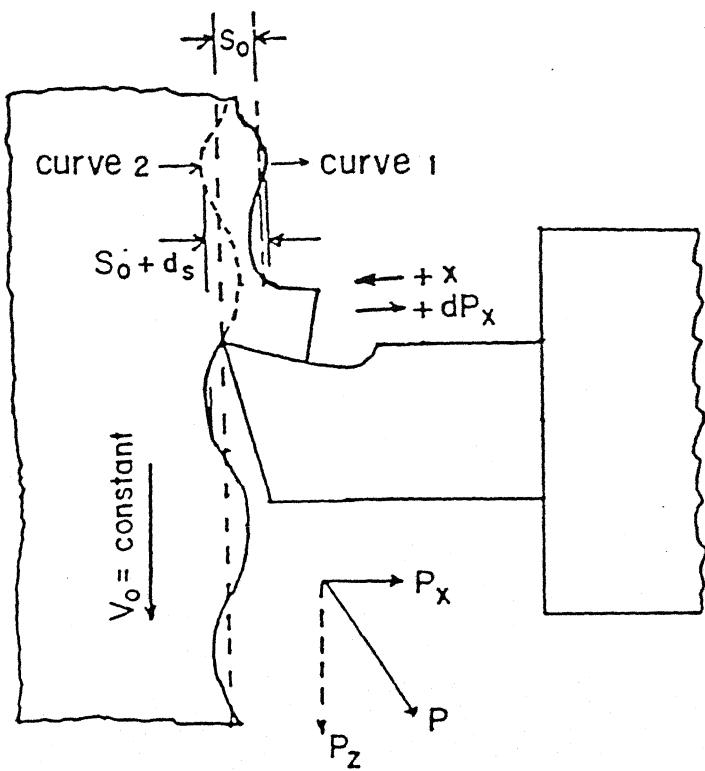
Writing  $m_2 = \lambda m_1$  we get

$$\omega_2/\omega_1 = \frac{1}{\sqrt{1+\lambda}} \quad (2.50)$$

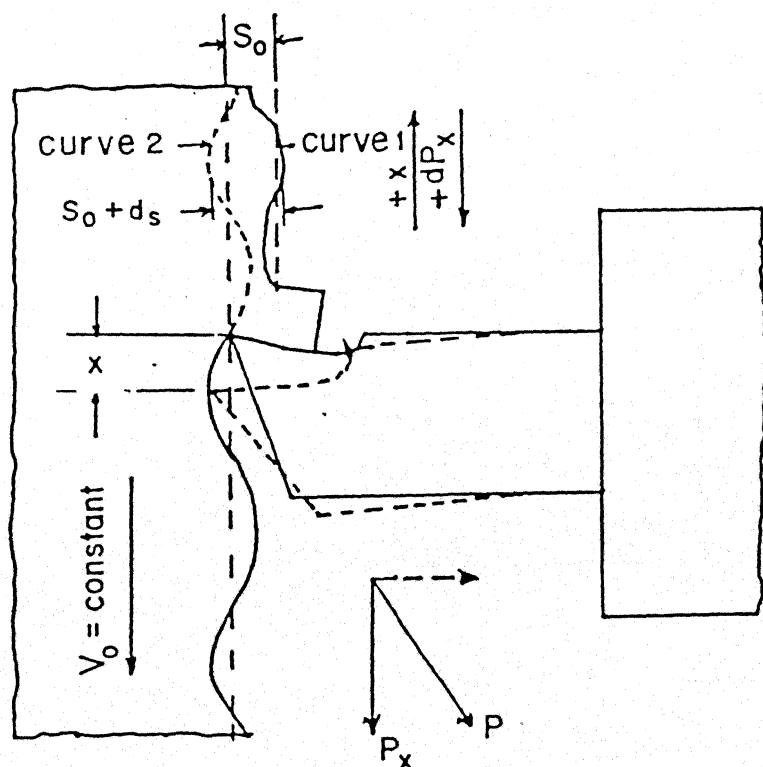
Substituting Eq. (2.50) in Eq. (2.49) we get

$$B^2 = \frac{\mu^2 \left( \frac{K_1}{K_a} \right)^2 \sin^2(\omega / N)}{1 + \frac{K_1}{K_a} (1 - \mu \cos \omega / N)} \left\{ 2 + \frac{K_1 (1 - \mu \cos \omega / N)}{K_a} \right\} (1 + \lambda) \quad (2.51)$$

Comparing Eq. (2.51) and Eq. (2.31), it is found that  $B^2$  value of Eq. (2.51) which incorporates the vibration absorber is more. Therefore the value of  $Q_e$  obtained from Eq.(2.29) will be more than  $Q_1$  obtained from Eq. (2.46) for the same value of  $K_r/K_a N$ . Hence by adopting the above absorber, the effective amplification of the main mass has reduced thereby giving better chatter free operation.

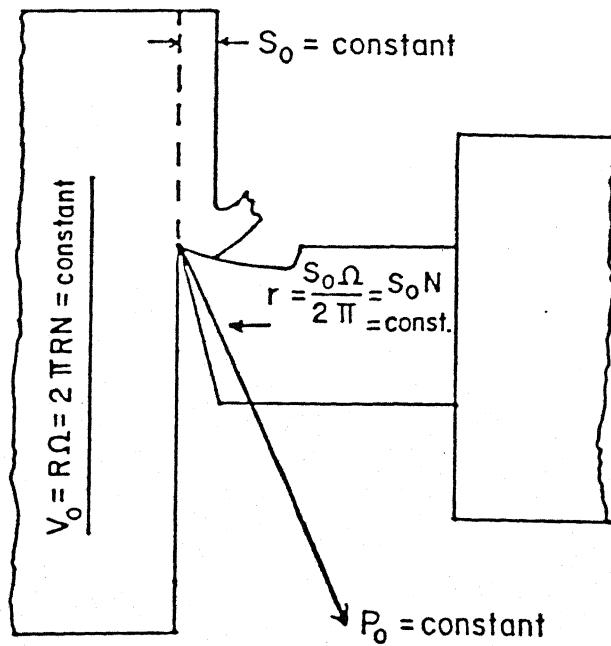


TYPE A CHATTER

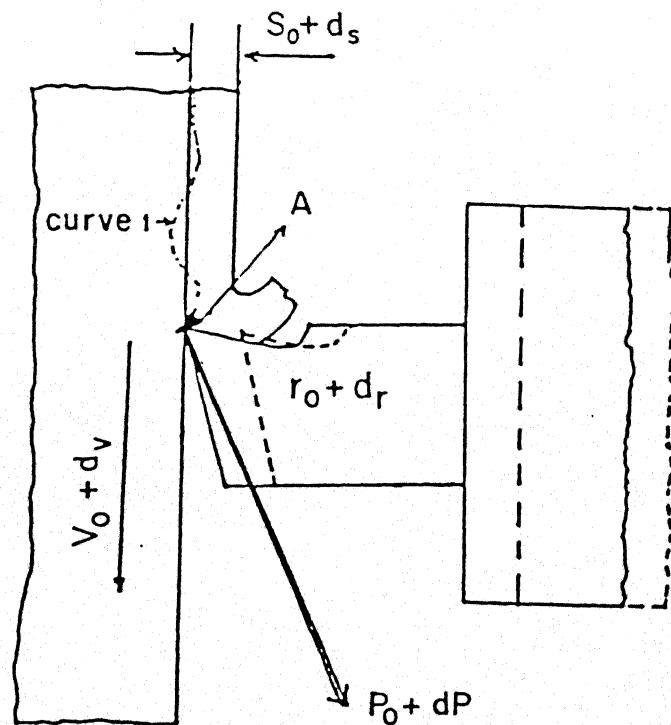


TYPE B CHATTER

Fig. 2.1 Types of chatter



(a) STATIC PROCESS



(b) DYNAMIC PROCESS

Fig. 2.2 DIFFERENCE BETWEEN STATIC AND DYNAMIC CUTTING PROCESS

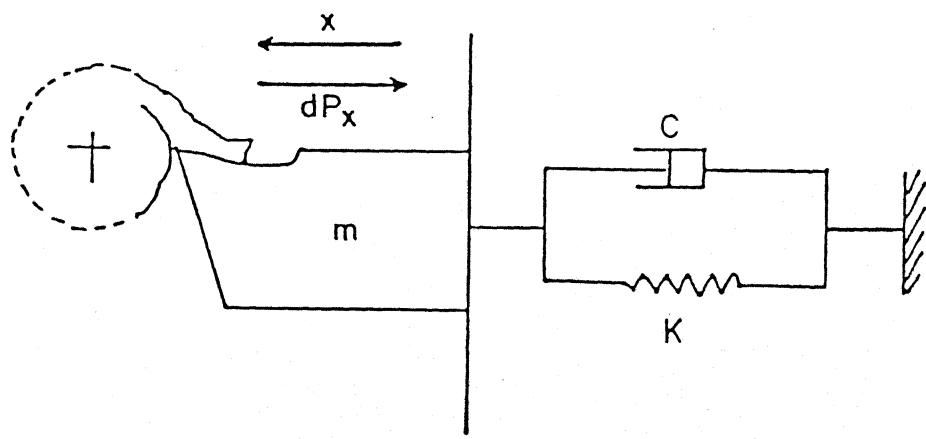


Fig 2.3 Work piece- Tool system under regenerative chatter of type A.

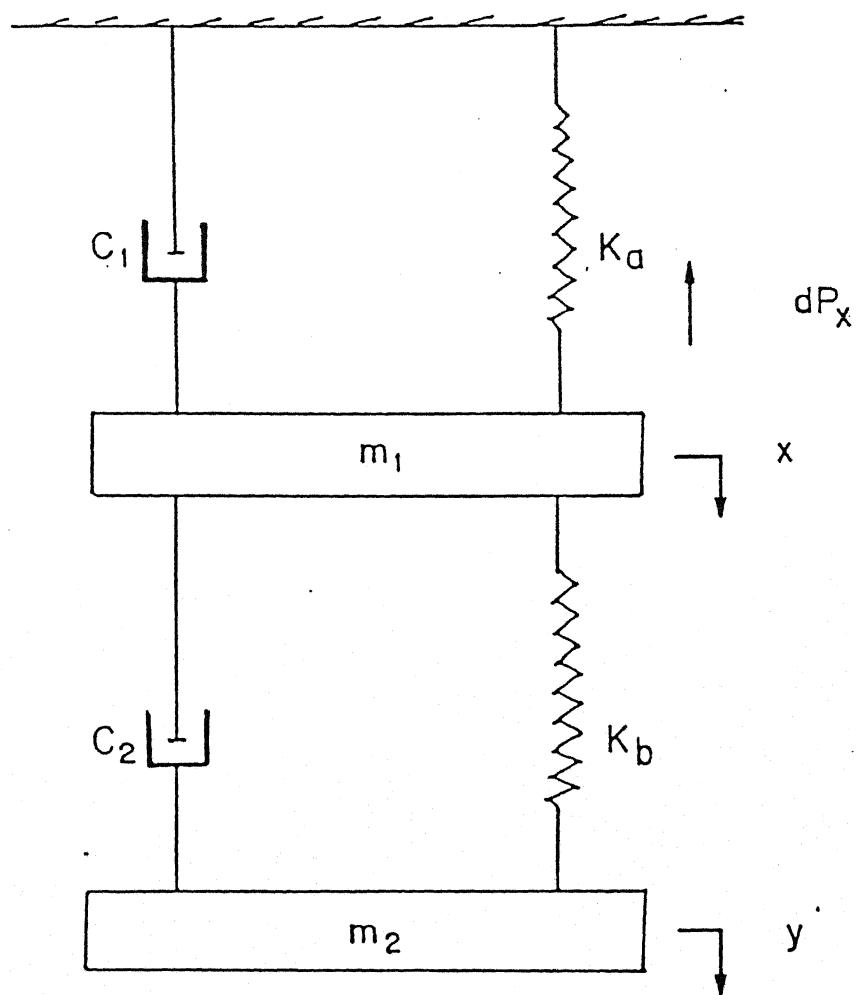


Fig. 2.4 Tuned vibration absorber

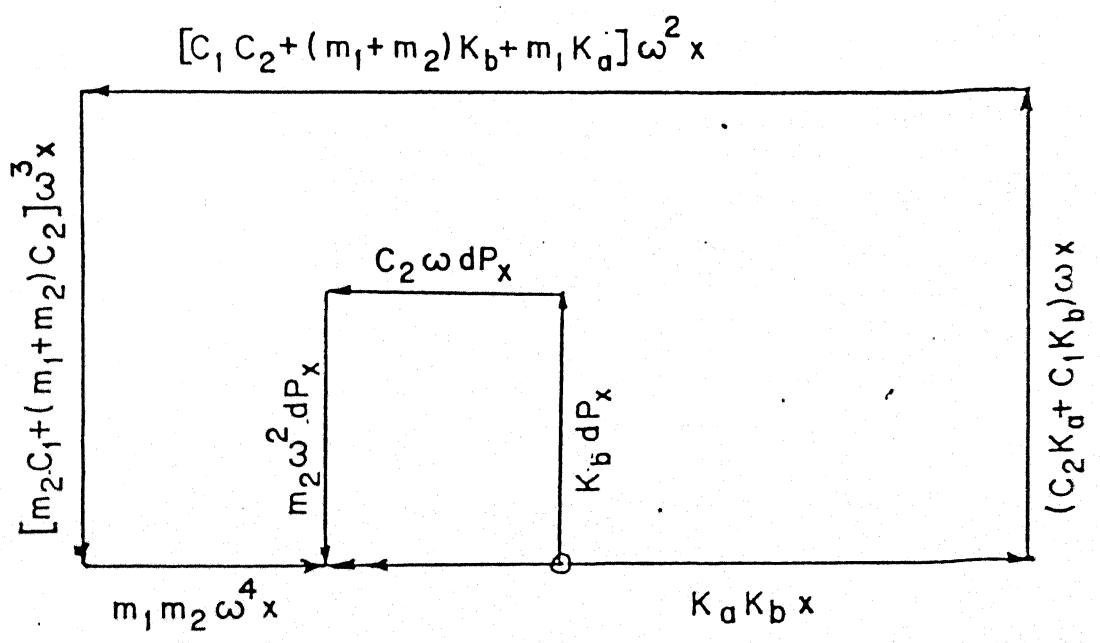


Fig. 2.5 Vector representation of equation 2.40

## CHAPTER III

### 3.1 DESCRIPTION OF THE SETUP :

A schematic diagram of the experimental set is shown in Fig. 3.1.

#### 3.1a Design of the absorber :

The absorber is essentially a flat spring (1) resting on two movable supports (3), which are essentially nuts, attached to the ends of the side plates (6). A screw connected (2) with a step is mounted on the side plates (6). The screw has a left hand thread cut on one half and a right hand thread on the other. Thus, when the screw is rotated, the supports (3) move inward or outward simultaneously depending upon the direction of rotation of the screw. The need for having variable supports is that, by moving the supports the stiffness of the plate (i.e. the spring) can be changed and can be tuned according to the cutting conditions. The design considerations taken for the plate are :

- (i) It should be capable of providing the desired stiffness under various cutting condition.
- (ii) There should be no plastic deformation, that is it should regain its original flat shape after the load has been released.

With these conditions in mind, a flat spring steel plate of 3 mm thickness and 32 mm width is used as an absorber.

It is essential that the flat spring should be resting on the supports evenly and during the axial movements of the supports, there should not be a gap created between the supports and the spring at any point. To achieve this, four ball bearings are attached to the movable supports. The height of the bearings can be adjusted so that at any instance, the flat spring is evenly supported on four ball bearings and load distribution during cutting operation is even.

### 3.1b Tool Holder :

The tool holder has to satisfy certain requirements. It should be able to withstand all the forces acting on the tool. There must be a provision for the excitation forces to be transmitted to the tool. However, the tool holder must not be too weak so as to deflect excessively under the action of the cutting forces. Furthermore, top and side face of the tool holder is evenly cut and smoothened for proper placement of the accelerometer. A magnetic plug having a groove for a mounting stud is placed on the top and side face for recording radial and tangential vibrational amplitudes respectively. The vibration accelerometer, is mounted at the other end of the stud. The weight of the absorber can generally only be some small fraction of the weight of the main mass [17]. That is why,

the weight of the tool holder block is approximately ten times more than the absorber to add reason related to their natural frequency.

The tool holding block is made of mild steel. It is supported about the central rod of the conventional tool post base. To accommodate the tool in the holder, an attachment, made of brass (10), is used. The circular attachment has a square hole to accommodate the tool. A plate (8) with a screw is attached to the tool holder to transmit the vibrations from tool to the absorber.

### 3.1c Measuring Instruments :

In order to set, measure and record the signals under actual machining conditions, the following instruments were used.

- (i) **Vibration Accelerometers** : Endevco model 22I5F accelerometers were used for the experiments. These accelerometers incorporate Piezite element Type P-8 for greater sensitivity. Essential specifications are mentioned in annexure.
- (ii) **Amplifier** : Endevco model 2616 B input amplifier were used for the experiments. This amplifier is recommended by the company for use with above accelerometers. It is designed to provide a very high input resistance of 1000 megaohms and gains of 1, 3 and 10 into an output load of 2500 ohms or more. The very high input resistance assures adequate low frequency response.

Standardization of the output signal is accomplished by means of variable shunt capacity across the input. For the present experiment, the gain switch was set at 10. The amplifier was connected to Endevco model 2622 power supply as recommended by the company.

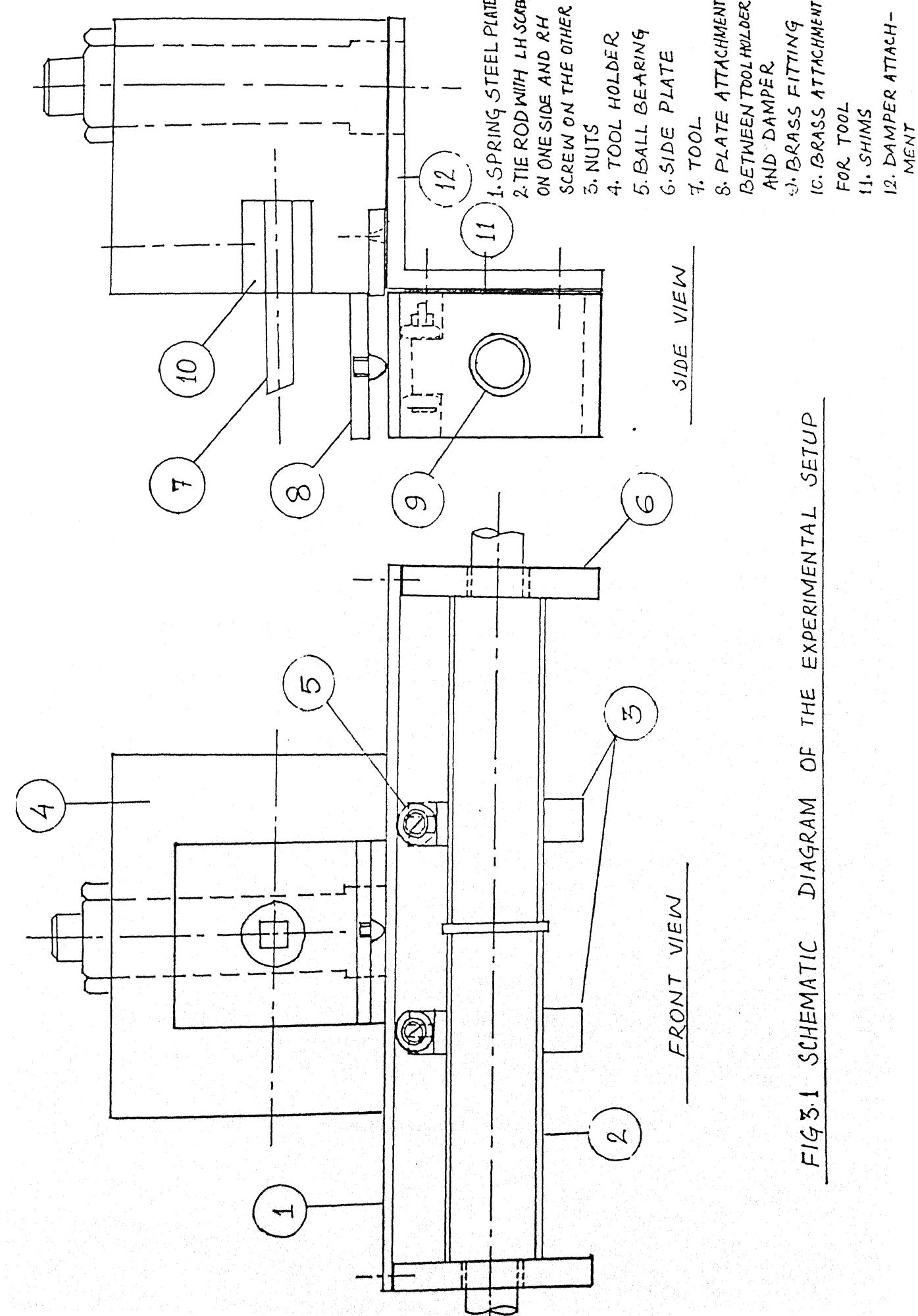
- (iii) Oscilloscope : THS 710 Tekscope instrument was used for the experiments. the Tekscope has a two channel oscilloscope and a digital multimeter in a rugged, hand held package. The Tekscope has a facility to store the waveforms and data. The Tekscope is fully programmable through RS-232 communication port which also helps to download the waveforms and make hard copies. Vibration signals from the machining zone sensed by the accelerometer were fed through the amplifier to the Tekscope. The Tekscope was later connected to a laser jet printer to download the waveforms and make hard copies.

### **3.2 PROCEDURE**

A series of experiments were conducted to determine the performance of the absorber system. Experiments were conducted at variable feeds and speeds.

The workpiece, a mild steel rod, was clamped in a three jaw chuck and has supported at the tailstock by revolving centres. The workpiece was first rough turned to remove the ovality as well as hard scale. This rough turned workpiece was then turned with the tool in the presence of the absorber.

The job was first turned in the absence of the absorber upto a certain distance. The absorber was then incorporated and the cutting was commenced. Readings were taken at four gap lengths of 20 mm, 40 mm, 60 mm and 80 mm. The distance between the nuts are chosen arbitrarily. The experiments had to be carried out at each speed for all the feed values chosen, both with the without the absorber. Also readings were recorded in both radial as well as tangential directions. The oscilloscope used for recording the vibration amplitudes had a digital multimeter which could give peak to peak as well as average value of the recorded vibration amplitudes. But unfortunately it was not working. The other alternative was to take a print out of the recorded vibration amplitude and then find the average value. Also the oscilloscope had a memory capacity to store only ten waveforms at any time. With these limitations, it was not possible to record the vibration amplitude at each turn of the screw. That is why, the scope of the experiments were restricted to only above mentioned gap lengths. While changing the gap length, the machine was stopped and the gap length was adjusted by rotating the screw until proper gap length was achieved. The gap lengths were selected arbitrarily for the first set and were kept constant for all the subsequent set of experiments. The vibrations of the tool holder was recorded in all cases in the oscilloscope. The amplitudes of the vibration signatures were recorded in millivolts. The values of amplitudes obtained for the cutting conditions under which the experiments were conducted are shown in Table 4.1 to Table 4.3. Measurements were taken in both tangential and radial directions.



**FIG 3.1 SCHEMATIC DIAGRAM OF THE EXPERIMENTAL SETUP**

# CHAPTER IV

## EXPERIMENTAL RESULTS AND DISCUSSIONS

### **4.1 RESULTS :**

Table 4.1 shows vibration amplitudes recorded in millivolts along radial and tangential directions for feed =0.05 mm/rev, which is constant, against varying rotational speed. Table 4.2 and Table 4.3 show similar readings but for feed=0.1 mm/rev and feed=0.2 mm/rev respectively.

From Table 4.1 to Table 4.3 it becomes evident that the present absorber system is capable of improving the vibration characteristics under the given cutting conditions. The level of improvement depends upon the cutting conditions, which has been expressed by a co-efficient,  $K_i$  as

$$K_i = \frac{\text{Average chatter amplitude in millivolt without absorber}}{\text{Average chatter amplitude in millivolt with absorber}}$$

The values of the improvement co-efficient for different cutting conditions are listed in Table 4.4 to Table 4.6. The values show a reasonable improvement in reducing the vibration within the range of experiments conducted. Furthermore, Figures 4.1-4.6

indicate the improvement co-efficient as a function of rotational speed for various gap lengths and a constant feed. The absorber was required to be tuned by changing the gap length between the nuts for a particular rotational speed and feed combination. With the tuning of the absorber, the chatter amplitude was found to vary. The figures above highlight the best tuning condition for a particular rotational speed and feed combination. The figures are represented in terms of improvement co-efficient.

Table 4.7 and Figures 4.1-4.2 show that for feed=0.05 mm/rev, as the speed increases, there is a decline in improvement co-efficient for a particular tuning condition of the absorber. The best results have been obtained in both radial and tangential directions as the absorber is tuned to the gap length of 40 mm when the rotational speed is 57 rpm. In the selected tuning range, the improvement co-efficient dips at rotational speed of 90 rpm. Therefore, for feed=0.05 mm/rev, the absorber should not be used for rotational speed of 90 rpm.

Similarly Table 4.8 and Figures 4.3-4.4 show that the best rotational speed is 90 rpm with the gap length between the nuts kept at 40 mm.

Similarly Table 4.9 and Figures 4.5-4.6 show that with the absorber tuned to the gap length of 40 mm, the improvement co-efficient shows a rising trend as the speed is increased with feed remaining constant.

It has already been highlighted in the procedure why readings could not be taken at every turn of the screw. Therefore, the results so obtained only gives an indication of the rotational speed and feed combination which will give better chatter suppress cutting conditions. Proper adjustment of the gap length between the nuts around these operating conditions may indicate further improvement in  $K_i$ .

#### **4.2 DISCUSSION OF RESULTS :**

It has been already discussed in section 2.1(a) that under dynamic cutting condition, there are three parameters which may vary independently of each other and these are chip thickness  $S$ , feed rate  $r$ , and cutting speed  $V$ . According to equation 2.10 of section 2.1(a), the variation of these parameters with their respective co-efficients namely, chip thickness co-efficient  $K_1$ , penetration rate co-efficient  $K_r$ , and cutting speed co-efficient  $K_v$ , determine the cutting force variation  $dP$  acting on the tool. The influence of all these three parameters together on  $dP$  has been analysed as per equation 2.10. The experimental determination of the coefficient  $K_1$  is difficult. Also we have assumed at section 2.1(c) that  $d\Omega=0$  and therefore the significance of cutting speed coefficient,  $K_v$ , has not been considered. Even if  $K_r$  can be calculated experimentally, in the absence of  $K_1$  it is difficult to say whether  $K_1$  or  $K_r$  has greater influence of  $dP$ .

For the purpose of present study two assumptions have been made. They are :

- (a) The chatter is of Type A as specified in section 2.1 and (b) There is no variation in cutting speed. Section 2.1(c) equation 2.22 and 2.23 gives the stability conditions for

the mode of vibration under study. Furthermore, these equations show the effect on stability of various changes in the cutting conditions. Owing to the transcendental nature of these two equations, their direct practical application is extremely difficult. It can be made easier by presenting the equations graphically in the form of stability charts. Such charts show whether the mode of vibration studied is stable or unstable at a specific rotational speed, or if it lies at the stability threshold. The transition stage between the stable and unstable condition is termed as the stability threshold.

The results obtained in section 4.1 are practical manifestations of the stability criteria studied in section 2.1(c) and 2.1(d). It has not been presented in the form of a usual stability chart because the experimental value of  $K_1$  and  $K_r$  are not available. In this way, only general conclusions can be drawn which are applicable to all cutting processes.

It has already been found out (equation 2.37 of section 2.1(c)) that for every rotational speed  $N$ , there is a minimum effective amplification factor,  $Q_m$ , and when  $Q_e$  is less than  $Q_m$ , the system is stable. This  $Q_m$  curve divides the stability chart into two regions, namely the region of unconditional stability ( $Q_e < Q_m$ ) and the region of conditional stability ( $Q_e > Q_m$ ). All the points in the first region correspond to chatter free cutting. Points in the region of conditional stability can correspond either to chatter free cutting or to cutting accompanied by chatter, depending on whether the points in question occupy a stable or an unstable rotational speed range. It can be noticed that the form of the  $Q_m$  curve given by equation 2.37 is governed principally by the sign and value of the ratio of

penetration co-efficient by stiffness ( $K_r/K$ ). Figure 4.7 shows  $Q_m$  curve for three characteristic values of  $K_r/K$  as given below.

For  $K_r/K = 0$ ;  $Q_m = Q_{mo} = \text{constant}$ ; it runs parallel to abscissa.

For  $K_r/K > 0$ ;  $Q_m = \infty$  when  $N/\omega_0 = K_r Q_m / K$

$$Q_m = Q_{mo} \text{ when } N/\omega_0 = \infty$$

For  $K_r/K < 0$ ;  $Q_m = 0$  when  $N/\omega_0 = 0$

$$Q_m = Q_{mo} \text{ when } N/\omega_0 = \infty$$

The experimental results obtained in the present study clearly indicate that it falls in the second region of conditional stability. That is why only at certain speeds improvement in chatter amplitude was observed. The general conclusion regarding the avoidance of instability are :

- (a)  $Q_e$  should be as small as possible
- (b)  $Q_{mo}$  line should be as high as possible.

#### 4.3 CONCLUSION :

A passive absorber for controlling chatter in lathe tools has been designed based on the principles of dynamic vibration absorbers and tested experimentally.

From a comparison of the theoretical analysis and actual results it was observed that the predictions of the theory tally with the experiments as far as the nature of predictions is concerned. For a given set of cutting conditions, there exists a particular tuning at which the absorber effectiveness is maximum as predicted by theory at equations 2.50 & 2.51 and this has been achieved during experiments. However, the effective dynamic cutting stiffness for a given set of cutting conditions could be more accurately predicted had the system parameters like stiffness of the tool holders in the principal direction, its natural frequency and damping etc. are known exactly.

#### **4.4 Suggestions for future work :**

In order to achieve significant improvement, continuous tuning of the absorber during the cutting operation is essential. Therefore to make the absorber capable of controlling chatter over a wide range of frequencies, it is necessary that the absorber be converted into an active control one with servo feedback system. The servo feedback system through proper logic should automatically adjust the gap between the spring supports for a particular cutting condition so as to achieve the highest improvement coefficient parameter .

**FEED = 0.05 MM/REV (CONSTANT)**

RPM	TANGENTIAL VIBRATION IN MILLIVOLTS		RADIAL VIBRATION IN MILLIVOLTS		GAP BETWEEN NUTS IN MM
	WITH-OUT DAM-PER	WITH DAM-PER	WITH-OUT DAM-PER	WITH DAM-PER	
200	81	71	125	70	20
		95		68	40
		69		81	60
		93		65	80
90	43	56	63	49	20
		76		51	40
		39		78	60
		54		62	80
57	52	25	79	35	20
		26		31	40
		37		53	60
		44		64	80

**Table 4.1**

**FEED = 0.1 MM/REV (CONSTANT)**

RPM	TANGENTIAL VIBRATION IN MILLIVOLTS		RADIAL VIBRATION IN MILLIVOLTS		GAP BETWEEN NUTS IN MM
	WITH-OUT DAM-PER	WITH DAM-PER	WITH-OUT DAM-PER	WITH DAM-PER	
200	85	74	145	101	20
		92		119	40
		78		123	60
		81		113	80
90	79	48	71	64	20
		43		54	40
		49		70	60
		60		46	80
57	46	30	49	36	20
		33		33	40
		37		43	60
		57		68	80

**Table 4.2**

**FEED = 0.2 MM/REV (CONSTANT)**

RPM	TANGENTIAL VIBRATION IN MILLIVOLTS		RADIAL VIBRATION IN MILLIVOLTS		GAP BETWEEN NUTS IN MM
	WITH-OUT DAM-PER	WITH DAM-PER	WITH-OUT DAM-PER	WITH DAM-PER	
200	109	96	111	93	20
		74		64	40
		117		133	60
		107		99	80
90	52	67	73	63	20
		36		48	40
		40		54	60
		64		90	80
57	49	31	47	45	20
		51		53	40
		33		71	60
		61		55	80

**Table 4.3**

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**FEED = 0.05 MM/REV (CONSTANT)**

RPM	(K <sub>i</sub> ) IMPROVEMENT CO-EFFICIENT		GAP LENGTH IN MM
	TANGEN-TIAL	RADIAL	
200	1.14	1.78	20
	0.85	1.83	40
	1.17	1.54	60
	0.87	1.92	80
90	0.76	1.29	20
	0.56	1.24	40
	1.10	0.80	60
	0.79	1.02	80
57	2.08	2.25	20
	2.0	2.54	40
	1.4	1.49	60
	1.18	1.23	80

**Table 4.4**

**FEED = 0.1 MM/REV (CONSTANT)**

RPM	(K <sub>i</sub> ) IMPROVEMENT CO-EFFICIENT		GAP LENGTH IN MM
	TANGENTIAL	RADIAL	
200	1.14	1.43	20
	0.92	1.21	40
	1.08	1.17	60
	1.04	1.28	80
90	1.64	1.10	20
	1.83	1.31	40
	1.61	1.01	60
	1.31	1.54	80
57	1.53	1.36	20
	1.39	1.48	40
	1.24	1.13	60
	0.80	0.72	80

**Table 4.5**

**FEED = 0.2 MM/REV (CONSTANT)**

RPM	(K <sub>i</sub> ) IMPROVEMENT CO-EFFICIENT		GAP LENGTH IN MM
	TANGEN-TIAL	RADIAL	
200	1.13	1.19	20
	1.47	1.73	40
	0.93	0.83	60
	1.01	1.12	80
90	0.77	1.14	20
	1.44	1.52	40
	1.30	1.33	60
	0.81	0.81	80
57	1.58	1.04	20
	0.96	0.88	40
	1.48	0.66	60
	0.80	0.85	80

**Table 4.6**

**FEED = 0.05 MM/REV (CONSTANT)**

RPM	(K <sub>i</sub> ) IMPROVEMENT CO-EFFICIENT		GAP LENGTH IN MM
	TANGENTIAL	RADIAL	
57	2.08	2.25	20
90	0.76	1.29	
200	1.14	1.78	
57	2.0	2.54	40
90	0.56	1.24	
200	0.85	1.83	
57	1.40	1.49	60
90	1.10	0.80	
200	1.17	1.54	
57	1.18	1.23	80
90	0.79	1.02	
200	0.87	1.97	

**Table 4.7 : Variation of rotational speed vs improvement co-efficient**

**FEED = 0.1 MM/REV (CONSTANT)**

RPM	(K <sub>i</sub> ) IMPROVEMENT CO-EFFICIENT		GAP LENGTH IN MM
	TANGEN-TIAL	RADIAL	
57	1.53	1.36	20
	1.64	1.10	
	1.14	1.43	
90	1.39	1.48	40
	1.83	1.31	
	0.92	1.21	
200	1.24	1.13	60
	1.61	1.01	
	1.08	1.17	
57	0.80	0.72	80
	1.31	1.54	
	1.04	1.28	

**Table 4.8: Variation of rotational speed vs improvement co-efficient**

**FEED = 0.2 MM/REV (CONSTANT)**

RPM	(K <sub>i</sub> ) IMPROVEMENT CO-EFFICIENT		GAP LENGTH IN MM
	TANGENTIAL	RADIAL	
57	1.58	1.04	20
90	0.77	1.14	
200	1.13	1.19	
57	0.96	0.88	40
90	1.44	1.52	
200	1.47	1.73	
57	1.48	0.66	60
90	1.30	1.33	
200	0.93	0.83	
57	0.80	0.85	80
90	0.81	0.81	
200	1.01	1.12	

**Table 4.9: Variation of rotational speed vs improvement co-efficient**

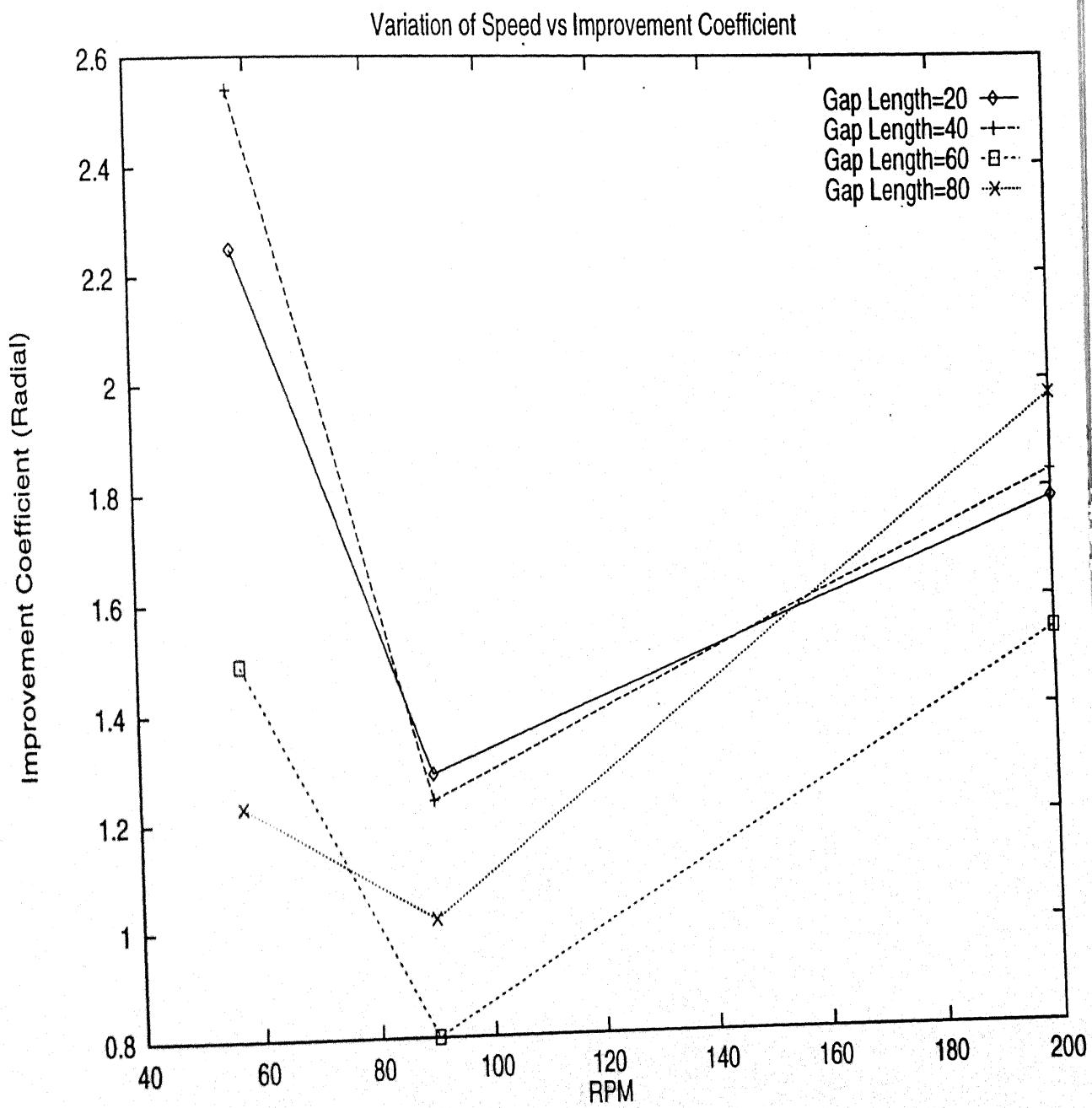


Figure 4.1: Feed 0.05 mm/rev. (Constant)

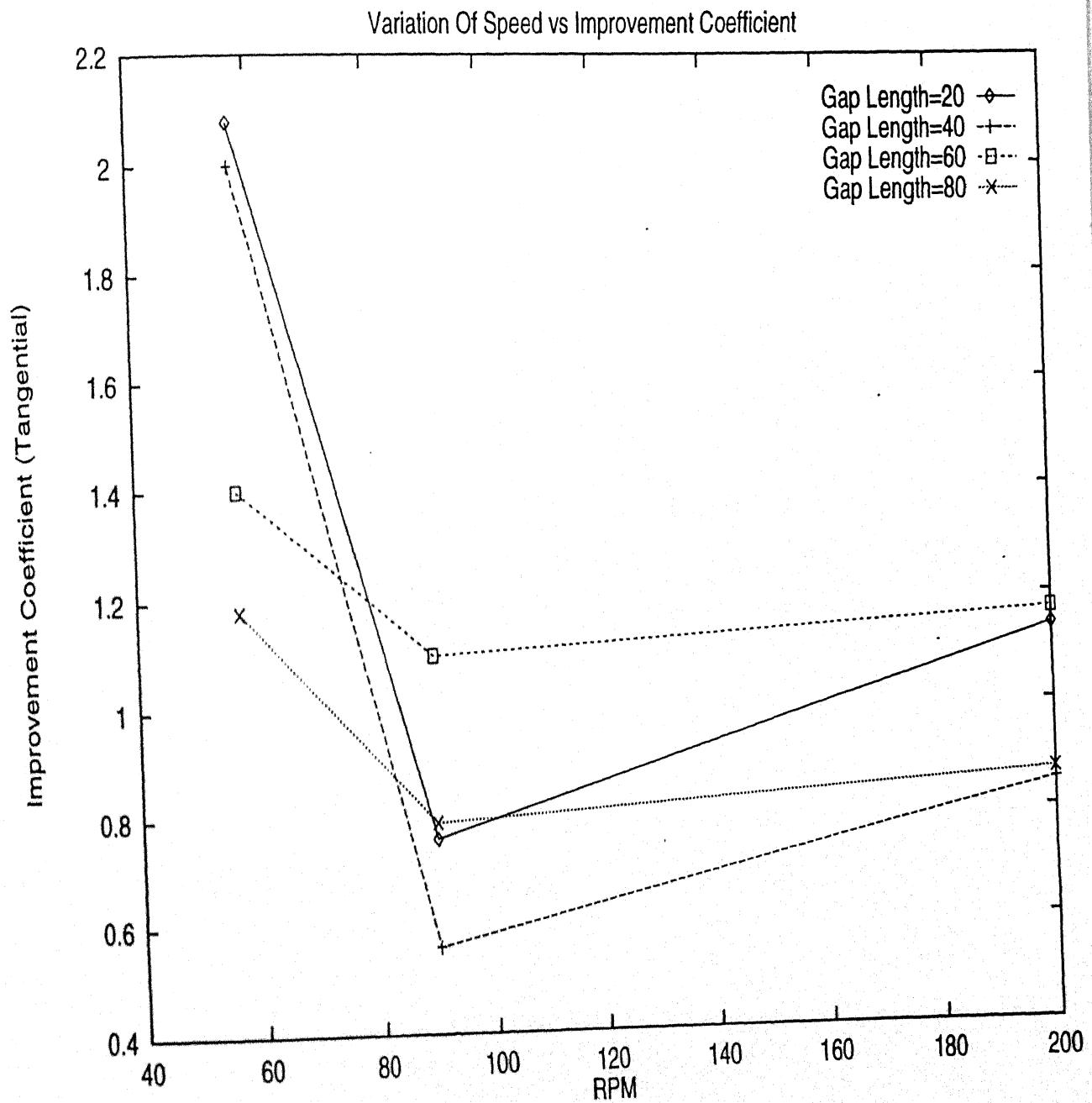


Figure 4.2: Feed 0.05 mm/rev. (Constant)

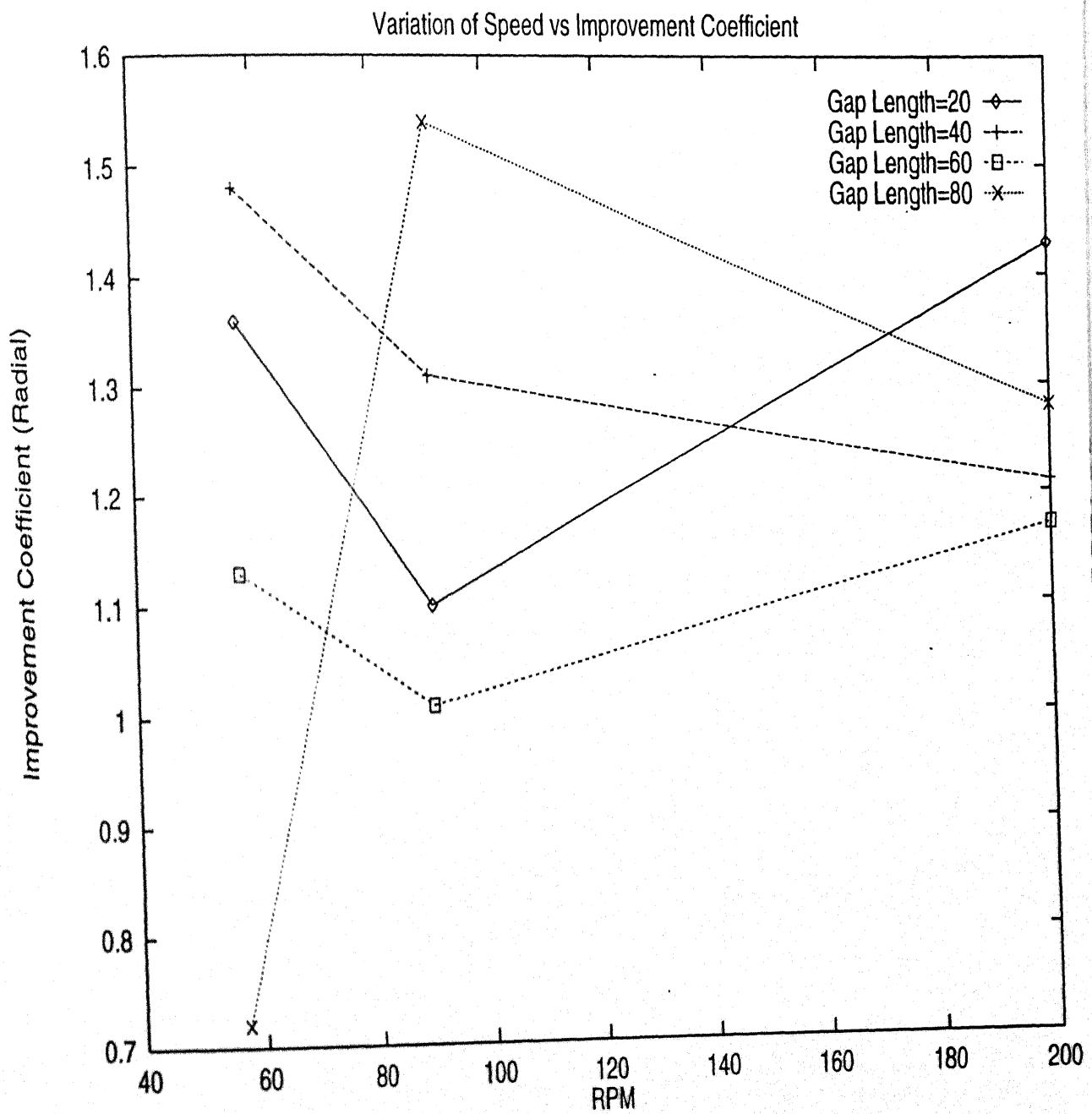


Figure 4.3: Feed 0.10 mm/rev. (Constant)

Variation of Speed vs Improvement Coefficient

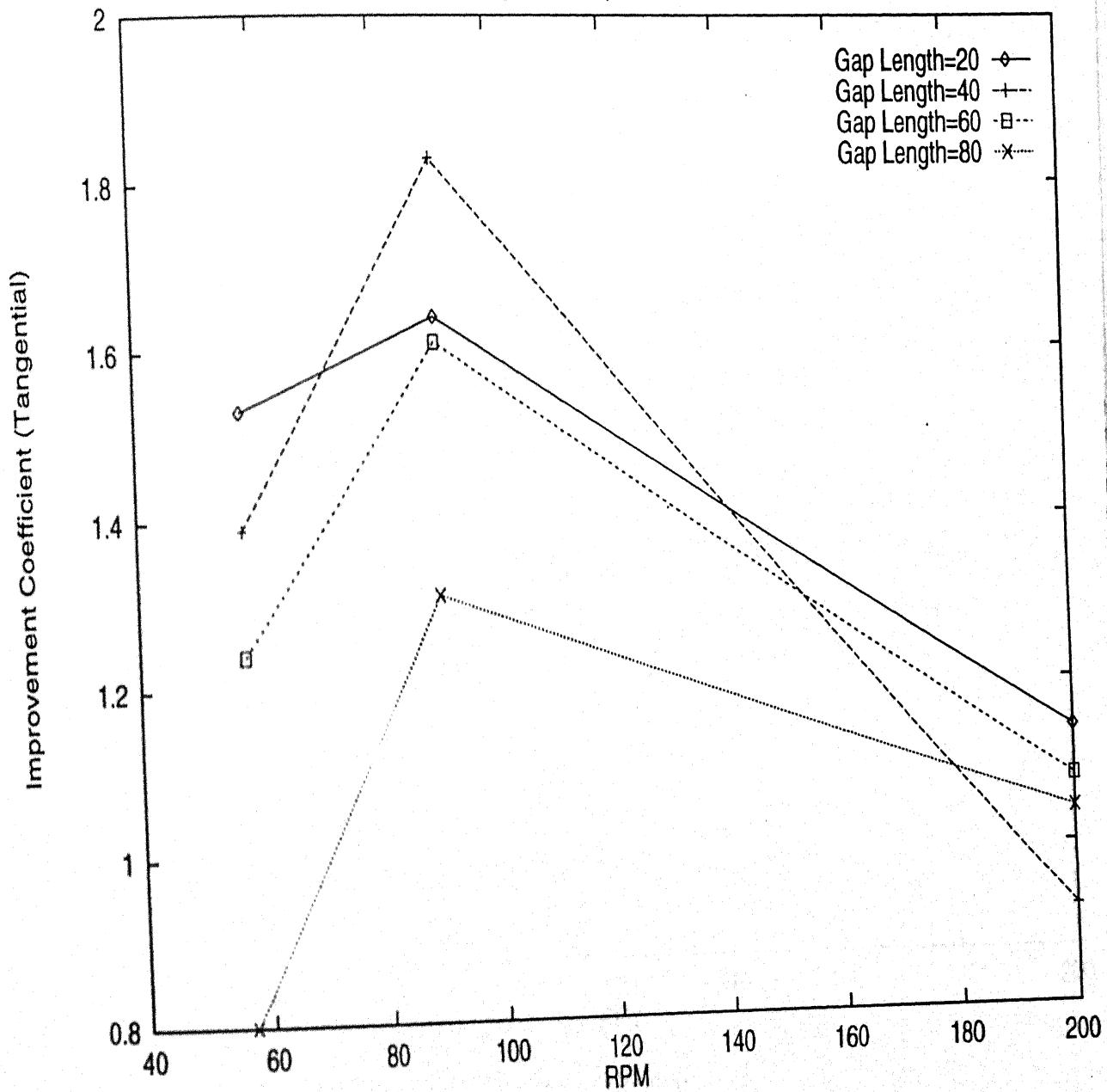


Figure 4A: Feed 0.10 mm/rev. (Constant)

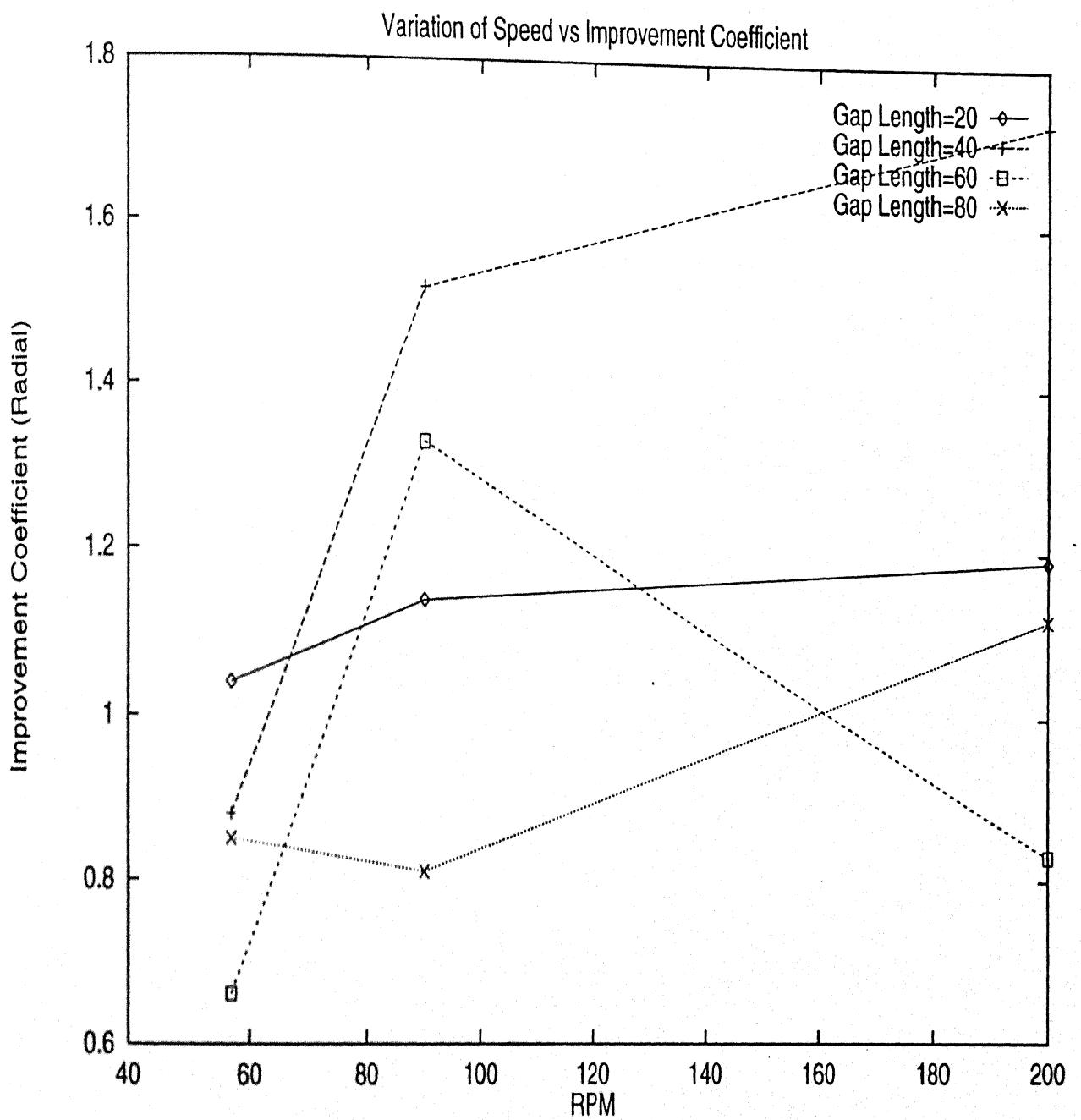


Figure 4.5: Feed 0.20 mm/rev. (Constant)

Variation of Speed vs Improvement Coefficient

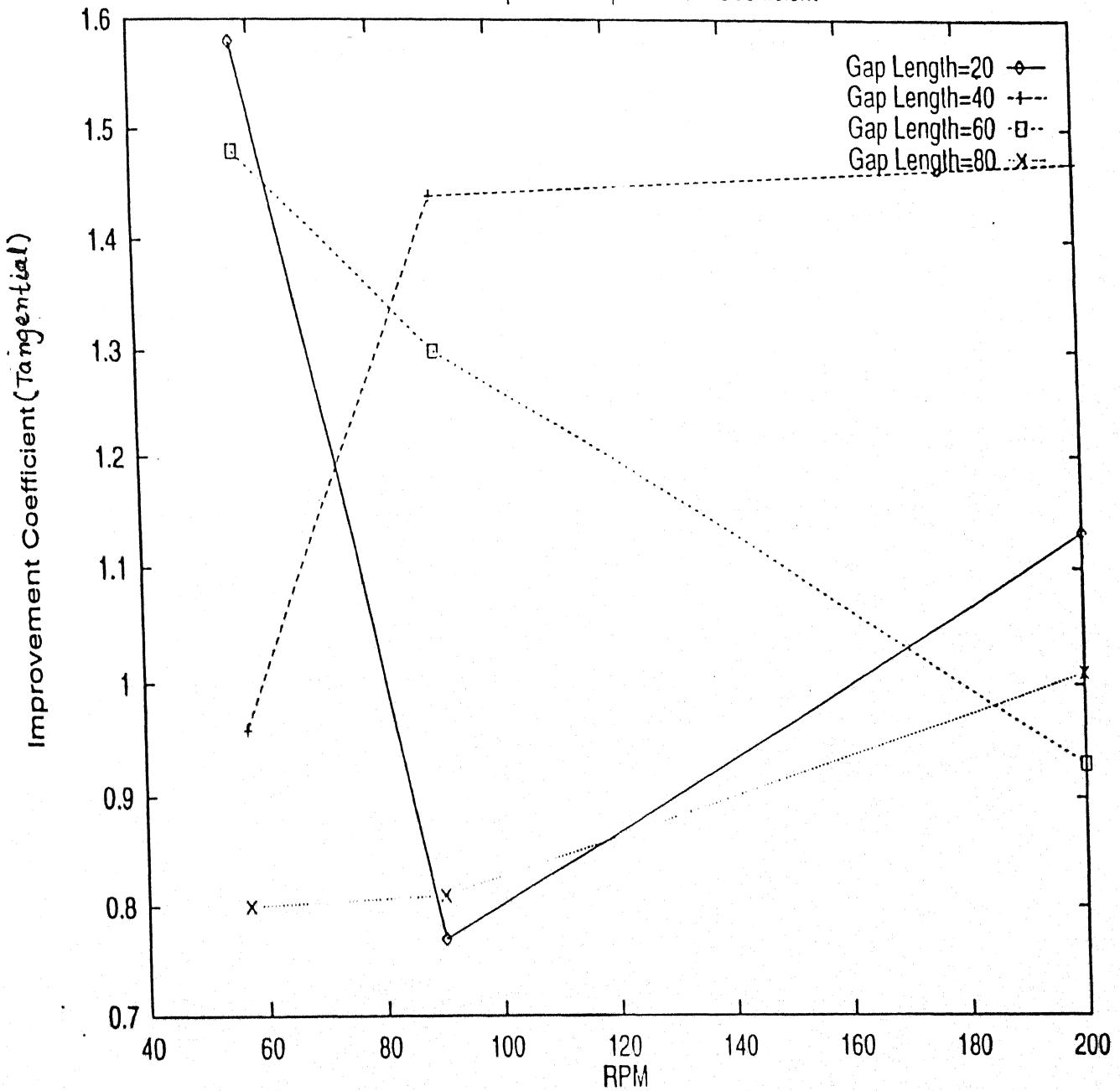


Figure 4.6: Feed 0.20 mm/rev. (Constant)

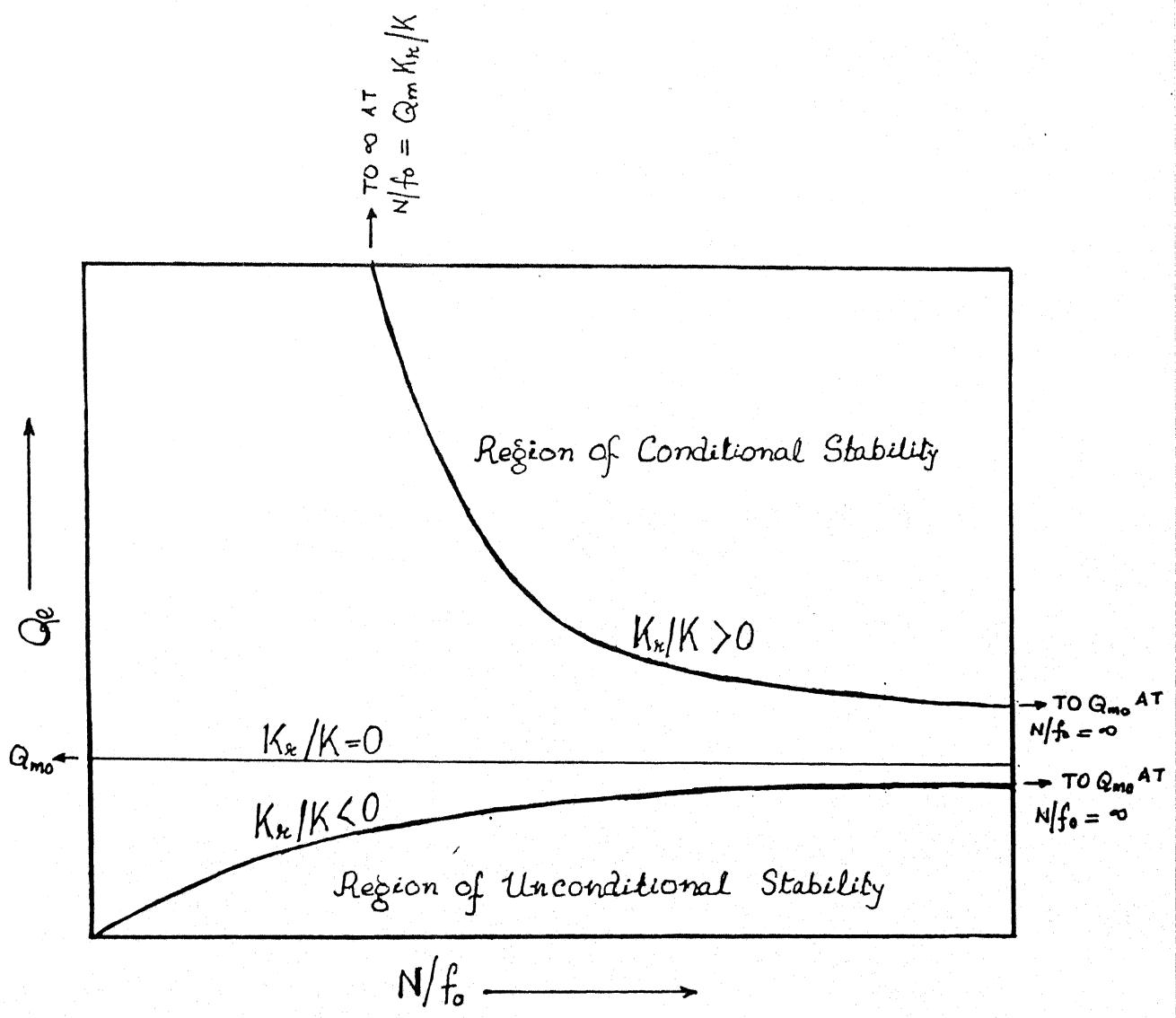


FIG. 4.7       $Q_m$  CURVE

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## **Annexure**

### **Specification for 2215F accelerometer**

Charge Sensitivity	150pk pC / pk g, nominal 120pk pC / pk g, minimum
Voltage Sensitivity	15pk mV / pkg, nominal
Transducer Capacitance	9500pC, $\pm 20\%$
Mounted Resonant Frequency	32,000Hz, $\pm 10\%$
Resistance	20,000M $\Omega$ , min. at R.T. 500M $\Omega$ , min. at +350°F
Frequency Response ( $\pm 5\%$ )	2 to 6000Hz with 100M $\Omega$ load. 5 to 6000Hz with 10M $\Omega$ load.
Transverse Sensitivity	3%, max., in any axis
Amplitude Linearity	$\pm 2\%$ , 0 to 1000g ( per ASA S2.2-1959)

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